2024 CFA® Exam Prep

SchweserNotes™

Fixed Income, Derivatives, and Alternative Investments

LEVEL II BOOK 4

KAPLAN SCHWESER

Book 4: Fixed Income, Derivatives, and Alternative Investments

SchweserNotes™ 2024

Level II CFA®

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SCHWESERNOTES™ 2024 LEVEL II CFA® BOOK 4: FIXED INCOME, DERIVATIVES, AND ALTERNATIVE INVESTMENTS

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ALTERNATIVE INVESTMENTS

Learning Outcome Statements (LOS)

25. The Term Structure and Interest Rate Dynamics

The candidate should be able to:

a. describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve.
b. describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping.
c. describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management.
d. describe the strategy of rolling down the yield curve.
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f. calculate and interpret the swap spread for a given maturity.
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28. Credit Analysis Models

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c. calculate the expected return on a bond given transition in its credit rating.
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29. Credit Default Swaps

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b. describe credit events and settlement protocols with respect to CDS.
c. explain the principles underlying and factors that influence the market's pricing of CDS.
d. describe the use of CDS to manage credit exposures and to express views regarding changes in the shape and/or level of the credit curve.
e. describe the use of CDS to take advantage of valuation disparities among separate markets, such as bonds, loans, equities, and equity-linked instruments.

30. Pricing and Valuation of Forward Commitments

The candidate should be able to:

a. describe how equity forwards and futures are priced, and calculate and interpret their no-arbitrage value.
b. describe the carry arbitrage model without underlying cashflows and with underlying cashflows.
c. describe how interest rate forwards and futures are priced, and calculate and interpret their no-arbitrage value.
d. describe how fixed-income forwards and futures are priced, and calculate and interpret their no-arbitrage value.
e. describe how interest rate swaps are priced, and calculate and interpret their no-arbitrage value.
f. describe how currency swaps are priced, and calculate and interpret their no-arbitrage value.
g. describe how equity swaps are priced, and calculate and interpret their no-arbitrage value.

31. Valuation of Contingent Claims

The candidate should be able to:

a. describe and interpret the binomial option valuation model and its component terms.
b. describe how the value of a European option can be analyzed as the present value of the option's expected payoff at expiration.
c. identify an arbitrage opportunity involving options and describe the related arbitrage.
d. calculate the no-arbitrage values of European and American options using a two-period binomial model.
e. calculate and interpret the value of an interest rate option using a two-period binomial model.
f. identify assumptions of the Black–Scholes–Merton option valuation model.
g. interpret the components of the Black–Scholes–Merton model as applied to call options in terms of a leveraged position in the underlying.

h. describe how the Black–Scholes–Merton model is used to value European options on equities and currencies.
i. describe how the Black model is used to value European options on futures.
j. describe how the Black model is used to value European interest rate options and European swaptions.
k. interpret each of the option Greeks.
l. describe how a delta hedge is executed.
m. describe the role of gamma risk in options trading.
n. define implied volatility and explain how it is used in options trading.

32. Introduction to Commodities and Commodity Derivatives

The candidate should be able to:

a. compare characteristics of commodity sectors.
b. compare the life cycle of commodity sectors from production through trading or consumption.
c. contrast the valuation of commodities with the valuation of equities and bonds.
d. describe types of participants in commodity futures markets.
e. analyze the relationship between spot prices and futures prices in markets in contango and markets in backwardation.
f. compare theories of commodity futures returns.
g. describe, calculate, and interpret the components of total return for a fully collateralized commodity futures contract.
h. contrast roll return in markets in contango and markets in backwardation.
i. describe how commodity swaps are used to obtain or modify exposure to commodities.
j. describe how the construction of commodity indexes affects index returns.

33. Real Estate Investments

The candidate should be able to:

a. compare the characteristics, classifications, principal risks, and basic forms of public and private real estate investments.
b. explain portfolio roles and economic value determinants of real estate investments.
c. discuss commercial property types, including their distinctive investment characteristics.
d. explain the due diligence process for both private and public equity real estate investment.
e. discuss real estate investment indexes, including their construction and potential biases.
f. discuss types of publicly traded real estate securities.
g. justify the use of net asset value per share (NAVPS) in valuation of publicly traded real estate securities and estimate NAVPS based on forecasted cash net operating income.
h. describe the use of funds from operations (FFO) and adjusted funds from operations (AFFO) in REIT valuation.
i. calculate and interpret the value of a REIT share using the net asset value, relative value (price-to-FFO and price-to-AFFO), and discounted cash flow approaches.
j. explain advantages and disadvantages of investing in real estate through publicly traded securities compared to private vehicles.

34. Hedge Fund Strategies

The candidate should be able to:

a. discuss how hedge fund strategies may be classified.
b. discuss investment characteristics, strategy implementation, and role in a portfolio of equity-related hedge fund strategies.
c. discuss investment characteristics, strategy implementation, and role in a portfolio of event-driven hedge fund strategies.
d. discuss investment characteristics, strategy implementation, and role in a portfolio of relative value hedge fund strategies.
e. discuss investment characteristics, strategy implementation, and role in a portfolio of opportunistic hedge fund strategies.
f. discuss investment characteristics, strategy implementation, and role in a portfolio of specialist hedge fund strategies.
g. discuss investment characteristics, strategy implementation, and role in a portfolio of multi-manager hedge fund strategies.
h. describe how factor models may be used to understand hedge fund risk exposures.
i. evaluate the impact of an allocation to a hedge fund strategy in a traditional investment portfolio.

READING 25

THE TERM STRUCTURE AND INTEREST RATE DYNAMICS

EXAM FOCUS

This topic review discusses the theories and implications of the term structure of interest rates. In addition to understanding the relationships between spot rates, forward rates, yield to maturity, and the shape of the yield curve, be sure you become familiar with concepts like the Z-spread, the TED spread and the MRR-OIS spread. Interpreting the shape of the yield curve in the context of the theories of the term structure of interest rates is always important for the exam. Also pay close attention to the concept of key rate duration.

INTRODUCTION

The financial markets both impact and are controlled by interest rates. Understanding the term structure of interest rates (i.e., the graph of interest rates at different maturities) is one key to understanding the performance of an economy. In this reading, we explain how and why the term structure changes over time.

Spot rates are the annualized market interest rates for a single payment to be received in the future. Generally, we use spot rates for government securities (risk-free) to generate the spot rate curve. Spot rates can be interpreted as the yields on zero-coupon bonds, and for this reason we sometimes refer to spot rates as *zero-coupon rates*. A **forward rate** is an interest rate (agreed to today) for a loan to be made at some future date.

PROFESSOR'S NOTE

While most of the LOS in this topic review have *describe* or *explain* as the command words, we will still delve into numerous calculations, as it is difficult to really understand some of these concepts without getting in to the mathematics behind them.

Video covering this content is available online.

MODULE 25.1: SPOT AND FORWARD RATES, PART 1

LOS 25.a: Describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve.

SPOT RATES

The price today of $1 par, zero-coupon bond is known as the discount factor, which we will call P_T. Because it is a zero-coupon bond, the spot interest rate is the yield to maturity of this payment, which we represent as S_T. The relationship between the discount factor P_T and the spot rate S_T for maturity T can be expressed as:

$$P_T = \frac{1}{(1 + S_T)^T}$$

The term structure of spot rates—the graph of the spot rate S_T versus the maturity T—is known as the **spot yield curve** or **spot curve**. The shape and level of the spot curve changes continuously with the market prices of bonds.

FORWARD RATES

The annualized interest rate on a loan to be initiated at a future period is called the **forward rate** for that period. The term structure of forward rates is called the **forward curve**. (Note that forward curves and spot curves are mathematically related—we can derive one from the other.)

We will use the following notation:

$f(j,k)$ = the annualized interest rate applicable on a k-year loan starting in j years.

t = 0 t = j t = j + k

$F_{(j,k)}$ = the forward price of a $1 par zero-coupon bond maturing at time $j+k$ delivered at time j.

$$F_{(j,k)} = \frac{1}{[1 + f(j,k)]^k}$$

YIELD TO MATURITY

As we've discussed, the **yield to maturity (YTM)** or yield of a zero-coupon bond with maturity T is the spot interest rate for a maturity of T. However, for a coupon bond, if the spot rate curve is not flat, the YTM will not be the same as the spot rate.

EXAMPLE: Spot rates and yield for a coupon bond

Compute the price and yield to maturity of a three-year, 4% annual-pay, $1,000 face value bond given the following spot rate curve: S_1 = 5%, S_2 = 6%, and S_3 = 7%.

Answer:

1. Calculate the price of the bond using the spot rate curve:

$$\text{Price} = \frac{40}{(1.05)} + \frac{40}{(1.06)^2} + \frac{1040}{(1.07)^3} = \$922.64$$

2. Calculate the yield to maturity (y_3):

N = 3; PV = –922.64; PMT = 40; FV = 1,000; CPT I/Y → 6.94

$y_3 = 6.94\%$

Note that the yield on a three-year bond is a weighted average of three spot rates, so in this case we would expect $S_1 < y_3 < S_3$. The yield to maturity y_3 is closest to S_3 because the par value dominates the value of the bond and therefore S_3 has the highest weight.

EXPECTED AND REALIZED RETURNS ON BONDS

Expected return is the ex-ante holding period return that a bond investor expects to earn.

The expected return will be equal to the bond's yield only when *all three* of the following are true:

- The bond is held to maturity.
- All payments (coupon and principal) are made on time and in full.
- All coupons are reinvested at the original YTM.

The second requirement implies that the bond is option-free and there is no default risk.

The last requirement, reinvesting coupons at the YTM, is the least realistic assumption. If the yield curve is not flat, the coupon payments will not be reinvested at the YTM and the expected return will differ from the yield.

Realized return on a bond refers to the actual return that the investor experiences over the investment's holding period. Realized return is based on actual reinvestment rates.

THE FORWARD PRICING MODEL

The **forward pricing model** values forward contracts based on arbitrage-free pricing.

Consider two investors.

Investor A purchases a \$1 face value, zero-coupon bond maturing in *j+k* years at a price of $P_{(j+k)}$.

Investor B enters into a *j*-year forward contract to purchase a \$1 face value, zero-coupon bond maturing in *k* years at a price of $F_{(j,k)}$. Investor B's cost today is the present value of the cost: $PV[F_{(j,k)}]$ or $P_jF_{(j,k)}$.

Because the \$1 cash flows at *j+k* are the same, these two investments should have the same price, which leads to the forward pricing model:

$$P_{(j+k)} = P_jF_{(j,k)}$$

Therefore:

$$F_{(j,k)} = \frac{P_{(j+k)}}{P_j}$$

EXAMPLE: Forward pricing

Calculate the forward price two years from now for a $1 par, zero-coupon, three-year bond given the following spot rates.

The two-year spot rate, S_2 = 4%.

The five-year spot rate, S_5 = 6%.

Answer:

Calculate discount factors P_j and $P_{(j+k)}$.

$$P_j = P_2 = 1 / (1 + 0.04)^2 = 0.9246$$
$$P_{(j+k)} = P_5 = 1 / (1 + 0.06)^5 = 0.7473$$

The forward price of a three-year bond in two years is represented as $F_{(2,3)}$

$$F_{(j,k)} = P_{(j+k)} / P_j$$
$$F_{(2,3)} = 0.7473 / 0.9246 = 0.8082$$

In other words, $0.8082 is the price agreed to today, to pay in two years, for a three-year bond that will pay $1 at maturity.

PROFESSOR'S NOTE

In the Derivatives portion of the curriculum, the forward price is computed as the future value (for *j* periods) of $P_{(j+k)}$. It gives the same result and can be verified using the data in the previous example by computing the future value of P_5 (i.e., compounding for two periods at S_2).

FV = $0.7473(1.04)^2$ = $0.8082.

The Forward Rate Model

The **forward rate model** relates forward and spot rates as follows:

$$[1 + S_{(j+k)}]^{(j+k)} = (1 + S_j)^j [1 + f(j,k)]^k$$

or

$$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$$

This model is useful because it illustrates how forward rates and spot rates are interrelated.

This equation suggests that the forward rate *f*(2,3) should make investors indifferent between buying a five-year zero-coupon bond versus buying a two-year zero-coupon bond and at maturity reinvesting the principal for three additional years.

EXAMPLE: Forward rates

Suppose that the two-year and five-year spot rates are S_2 = 4% and S_5 = 6%.

Calculate the implied three-year forward rate for a loan starting two years from now [i.e., $f(2,3)$].

Answer:

$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$
$[1 + f(2,3)]^3 = [1 + 0.06]^5 / [1 + 0.04]^2$
$f(2,3) = 7.35\%$

Note that the forward rate $f(2,3) > S_5$ because the yield curve is upward sloping.

If the yield curve is upward sloping, [i.e., $S_{(j+k)} > S_j$], then the forward rate corresponding to the period from *j* to *k* [i.e., $f(j,k)$] will be greater than the spot rate for maturity *j+k* [i.e., $S_{(j+k)}$]. The opposite is true if the curve is downward sloping.

LOS 25.b: Describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping.

A **par rate** is the yield to maturity of a bond trading at par. Par rates for bonds with different maturities make up the **par rate curve** or simply the **par curve**. By definition, the par rate will be equal to the coupon rate on the bond. Generally, par curve refers to the par rates for government or benchmark bonds.

By using a process called **bootstrapping**, spot rates or zero-coupon rates can be derived from the par curve. Bootstrapping involves using the output of one step as an input to the next step. We first recognize that (for annual-pay bonds) the one-year spot rate (S_1) is the same as the one-year par rate. We can then compute S_2 using S_1 as one of the inputs. Continuing the process, we can compute the three-year spot rate S_3 using S_1 and S_2 computed earlier. Let's clarify this with an example.

EXAMPLE: Bootstrapping spot rates

Given the following (annual-pay) par curve, compute the corresponding spot rate curve:

Maturity	Par Rate
1	1.00%
2	1.25%
3	1.50%

Answer:

S_1 = 1.00% (given directly).

If we discount each cash flow of the bond using its yield, we get the market price of the bond. Here, the market price is the par value. Consider the 2-year bond.

$$100 = \frac{1.25}{(1.0125)} + \frac{101.25}{(1.0125)^2}$$

Alternatively, we can also value the 2-year bond using spot rates:

$$100 = \frac{1.25}{(1+S_1)} + \frac{101.25}{(1+S_2)^2} = \frac{1.25}{(1.01)} + \frac{101.25}{(1+S_2)^2}$$

$$100 = 1.2376 + \frac{101.25}{(1+S_2)^2}$$

$$98.7624 = \frac{101.25}{(1+S_2)^2}$$

Multiplying both sides by $[(1 + S_2)^2 / 98.7624]$, we get $(1 + S_2)^2 = 1.0252$.

Taking square roots, we get $(1 + S_2) = 1.01252$. $S_2 = 0.01252$ or 1.252%

Similarly, $100 = \frac{1.50}{(1+S_1)} + \frac{1.50}{(1+S_2)^2} + \frac{101.50}{(1+S_3)^3}$

Using the values of S_1 and S_2 computed earlier,

$$100 = \frac{1.50}{(1.01)} + \frac{1.50}{(1.01252)^2} + \frac{101.50}{(1+S_3)^3}$$

$$100 = 2.9483 + \frac{101.50}{(1+S_3)^3}$$

$$97.0517 = \frac{101.50}{(1+S_3)^3}$$

$(1 + S_3)^3 = 1.0458$
$(1 + S_3) = 1.0151$ and hence $S_3 = 1.51\%$

MODULE QUIZ 25.1

1. When the yield curve is downward sloping, the forward curves are *most likely* to lie:
   A. above the spot curve.
   B. below the spot curve.
   C. either above or below the spot curve.
2. The model that equates buying a long-maturity zero-coupon bond to entering into a forward contract to buy a zero-coupon bond that matures at the same time is known as the:
   A. forward rate model.
   B. forward pricing model.
   C. forward arbitrage model.

Video covering this content is available online.

MODULE 25.2: SPOT AND FORWARD RATES, PART 2

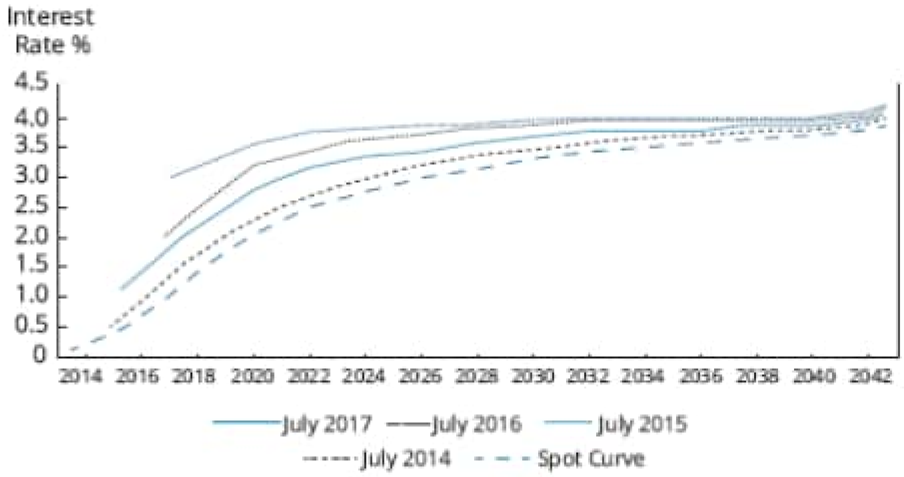
LOS 25.c: Describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management.

RELATIONSHIPS BETWEEN SPOT AND FORWARD RATES

For an upward-sloping spot curve, the forward rate rises as *j* increases. (For a downward-sloping yield curve, the forward rate declines as *j* increases.) For an upward-sloping spot curve, the forward curve will be above the spot curve as shown in Figure 25.1. Conversely, when the spot curve is downward sloping, the forward curve will be below it.

Figure 25.1 shows spot and forward curves as of July 2013. Because the spot yield curve is upward sloping, the forward curves lie above the spot curve.

Figure 25.1: Spot Curve and Forward Curves



Reproduced from Level II CFA Curriculum learning module "The Term Structure and Interest Rate Dynamics," Exhibit 2, with permission from CFA Institute.

From the forward rate model:

$$(1 + S_T)^T = (1 + S_1)[1 + f(1,T - 1)]^{(T-1)}$$

which can be expanded to:

$$(1 + S_T)^T = (1 + S_1)\,[1 + f(1,1)]\,[1 + f(2,1)]\,[1 + f(3,1)] \ldots [1 + f(T - 1,1)]$$

In other words, the spot rate for a long-maturity security will equal the geometric mean of the one period spot rate and a series of one-year forward rates.

Forward Price Evolution

If the future spot rates actually evolve as forecasted by the forward curve, the forward price will remain unchanged. Therefore, a change in the forward price indicates that the future spot rate(s) did not conform to the forward curve. When

spot rates turn out to be lower (higher) than implied by the forward curve, the forward price will increase (decrease). A trader expecting lower future spot rates (than implied by the current forward rates) would purchase the forward contract to profit from its appreciation.

For a bond investor, the return on a bond over a one-year horizon is always equal to the one-year risk-free rate *if the spot rates evolve as predicted by today's forward curve*. If the spot curve one year from today is not the same as that predicted by today's forward curve, the return over the one-year period will differ, with the return depending on the bond's maturity.

An active portfolio manager will try to outperform the overall bond market by predicting how the future spot rates will differ from those predicted by the current forward curve.

EXAMPLE: Spot rate evolution

Jane Dash, CFA, has collected benchmark spot rates as shown here:

Maturity	Spot Rate
1	3.00%
2	4.00%
3	5.00%

The expected spot rates at the end of one year are as follows:

Year	Expected Spot
1	5.01%
2	6.01%

Calculate the one-year holding period return of a:

1. 1-year zero-coupon bond.
2. 2-year zero-coupon bond.
3. 3-year zero-coupon bond.

Answer:

First, note that the expected spot rates provided just happen to be the forward rates implied by the current spot rate curve.

Recall that:

$$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$$

Hence:

$$[1 + f(1,1)]^1 = \frac{(1 + S_2)^2}{(1 + S_1)} = \frac{(1.04)^2}{(1.03)} \rightarrow f(1,1) = 0.0501 \text{ and}$$

$$[1 + f(1,2)]^2 = \frac{(1 + S_3)^3}{(1 + S_1)} = \frac{(1.05)^3}{(1.03)} \rightarrow f(1,2) = 0.0601$$

1. The price of a one-year zero-coupon bond given the one-year spot rate of 3% is 1 / (1.03) or 0.9709.
   After one year, the bond is at maturity and pays $1 regardless of the spot rates.

Hence the holding period return = $\left(\frac{1.00}{0.9709}\right) - 1 = 3\%$

2. The price of a two-year zero-coupon bond given the two-year spot rate of 4%:

$$P_2 = \frac{1}{(1 + S_2)^2} = \frac{1}{(1.04)^2} = 0.9246$$

After one year, the bond will have one year remaining to maturity, and based on a one-year expected spot rate of 5.01%, the bond's price will be 1 / (1.0501) = \$0.9523

Hence, the holding period return = $\left(\frac{0.9523}{0.9246}\right) - 1 = 3\%$

3. The price of three-year zero-coupon bond given the three-year spot rate of 5%:

$$P_3 = \frac{1}{(1 + S_3)^3} = \frac{1}{(1.05)^3} = 0.8638$$

After one year, the bond will have two years remaining to maturity.

Based on a two-year expected spot rate of 6.01%, the bond's price will be 1 / $(1.0601)^2$ = \$0.8898

Hence, the holding period return = $\left(\frac{0.8898}{0.8638}\right) - 1 = 3\%$

Hence, regardless of the maturity of the bond, the holding period return will be the one-year spot rate if the spot rates evolve consistent with the forward curve (as it existed when the trade was initiated).

If an investor believes that future spot rates will be lower than corresponding forward rates, then she will purchase bonds (at a presumably attractive price) because the market appears to be discounting future cash flows at "too high" of a discount rate.

LOS 25.d: Describe the strategy of rolling down the yield curve.

"ROLLING DOWN THE YIELD CURVE"

The most straightforward strategy for a bond investor is *maturity matching*—purchasing bonds that have a maturity equal to the investor's investment horizon.

However, with an upward-sloping interest rate term structure, investors seeking superior returns may pursue a strategy called **riding the yield curve** (also known as **rolling down the yield curve**). Under this strategy, an investor will purchase bonds with maturities longer than his investment horizon. In an upward-sloping yield curve, shorter maturity bonds have lower yields than longer maturity bonds. As the bond approaches maturity (i.e., rolls down the yield curve), it is valued using successively lower yields and, therefore, at successively higher prices.

If the yield curve remains unchanged over the investment horizon, riding the yield curve strategy will produce higher returns than a simple maturity matching strategy, increasing the total return of a bond portfolio. The greater the difference

between the forward rate and the spot rate, and the longer the maturity of the bond, the higher the total return.

Consider Figure 25.2, which shows a hypothetical upward-sloping yield curve and the price of a 3% annual-pay coupon bond (as a percentage of par).

Figure 25.2: Price of a 3%, Annual Pay Bond

Maturity	Yield	Price
5	3	100
10	3.5	95.84
15	4	88.88
20	4.5	80.49
25	5	71.81
30	5.5	63.67

A bond investor with an investment horizon of five years could purchase a bond maturing in five years and earn the 3% coupon but no capital gains (the bond can be currently purchased at par and will be redeemed at par at maturity). However, assuming no change in the yield curve over the investment horizon, the investor could instead purchase a 30-year bond for $63.67, hold it for five years, and sell it for $71.81, earning an additional return beyond the 3% coupon over the same period.

In the aftermath of the financial crisis of 2007–08, central banks kept short-term rates low, giving yield curves a steep upward slope. Many active managers took advantage by borrowing at short-term rates and buying long maturity bonds. The risk of such a leveraged strategy is the possibility of an increase in spot rates.

MODULE QUIZ 25.2

1. If the future spot rates are expected to be lower than the current forward rates for the same maturities, bonds are *most likely* to be:
   A. overvalued.
   B. undervalued.
   C. correctly valued.
2. The strategy of rolling down the yield curve is *most likely* to produce superior returns for a fixed income portfolio manager investing in bonds with maturity higher than the manager's investment horizon when the spot rate curve:
   A. is downward sloping.
   B. in the future matches that projected by today's forward curves.
   C. is upward sloping.

MODULE 25.3: THE SWAP RATE CURVE

Video covering this content is available online.

LOS 25.e: Explain the swap rate curve and why and how market participants use it in valuation.

THE SWAP RATE CURVE

In a plain vanilla interest rate swap, one party makes payments based on a fixed rate while the counterparty makes payments based on a floating rate. The fixed rate in an interest rate swap is called the **swap fixed rate** or **swap rate**.

If we consider how swap rates vary for various maturities, we get the **swap rate curve**, which has become an important interest-rate benchmark for credit markets.

Market participants prefer the swap rate curve as a benchmark interest rate curve rather than a government bond yield curve for the following reasons:

- Swap rates reflect the credit risk of commercial banks rather than the credit risk of governments.
- The swap market is not regulated by any government, which makes swap rates in different countries more comparable. (Government bond yield curves additionally reflect sovereign risk unique to each country.)
- The swap curve typically has yield quotes at many maturities, while the U.S. government bond yield curve has on-the-run issues trading at only a small number of maturities.

Wholesale banks that manage interest rate risk with swap contracts are more likely to use swap curves to value their assets and liabilities. Retail banks, on the other hand, are more likely to use a government bond yield curve.

Given a notional principal of $1 and a swap fixed rate SFR_T, the value of the fixed rate payments on a swap can be computed using the relevant (e.g., *MRR*) spot rate curve. For a given swap tenor *T*, we can solve for *SFR* in the following equation.

$$\sum_{t=1}^{T} \frac{SFR_T}{(1+S_t)^t} + \frac{1}{(1+S_T)^T} = 1$$

In the equation, *SFR* can be thought of as the coupon rate of a $1 par value bond given the underlying spot rate curve.

PROFESSOR'S NOTE

Prior to the 2023 curriculum, CFA Institute referenced LIBOR (the "London Interbank Offered Rate") as the underlying interest rate for derivative instruments. However, concerns about LIBOR being manipulated have led to LIBOR being replaced in recent years with other reference rates, such as the Federal Reserve Bank of New York's "SOFR" (Secured Overnight Financing Rate), and the Bank of England's "SONIA" (Sterling Overnight Index Average).

To reflect this change, the CFA curriculum now uses the generic term MRR ("market reference rate") to refer to these various LIBOR replacements.

EXAMPLE: Swap rate curve

Given the following MRR spot rate curve, compute the swap fixed rate for a tenor of 1, 2, and 3 years (i.e., compute the swap rate curve).

Maturity	Spot Rate
1	3.00%
2	4.00%
3	5.00%

Answer:

1. SFR_1 can be computed using the equation:

$$\frac{SFR_1}{(1+S_1)}+\frac{1}{(1+S_1)}=1$$

$$\frac{SFR_1}{(1.03)}+\frac{1}{(1.03)}=1 \rightarrow SFR_1=3.00\%$$

2. SFR_2 can be similarly computed:

$$\frac{SFR_2}{(1+S_1)}+\frac{SFR_2}{(1+S_2)^2}+\frac{1}{(1+S_2)^2}=1$$

$$\frac{SFR_2}{(1.03)}+\frac{SFR_2}{(1.04)^2}+\frac{1}{(1.04)^2}=1 \rightarrow SFR_2=3.98\%$$

3. Finally, SFR_3 can be computed as:

$$\frac{SFR_3}{(1+S_1)}+\frac{SFR_3}{(1+S_2)^2}+\frac{SFR_3}{(1+S_3)^3}+\frac{1}{(1+S_3)^3}=1$$

$$\frac{SFR_3}{(1.03)}+\frac{SFR_3}{(1.04)^2}+\frac{SFR_3}{(1.05)^3}+\frac{1}{(1.05)^3}=1 \rightarrow SFR_3=4.93\%$$

PROFESSOR'S NOTE

A different (and better) method of computing swap fixed rates is discussed in detail in the Derivatives portion of the curriculum.

MODULE QUIZ 25.3

1. Which of the following statements about the swap rate curve is *most accurate*?
   A. The swap rate reflects the interest rate for the floating-rate leg of an interest rate swap.
   B. Retail banks are more likely to use the swap rate curve as a benchmark than the government spot curve.
   C. Swap rates are comparable across different countries because the swap market is not controlled by governments.

MODULE 25.4: SPREAD MEASURES

Video covering this content is available online.

LOS 25.f: Calculate and interpret the swap spread for a given maturity.

Swap spread refers to the amount by which the swap rate exceeds the yield of a government bond with the same maturity.

$$\text{swap spread}_t = \text{swap rate}_t - \text{Treasury yield}_t$$

For example, if the fixed rate of a one-year fixed-for-floating MRR swap is 0.57% and the one-year Treasury is yielding 0.11%, the 1-year swap spread is 0.57% – 0.11% = 0.46%, or 46 bps.

Swap spreads are almost always positive, reflecting the lower credit risk of governments compared to the credit risk of surveyed banks that determines the swap rate.

The swap rate curve is arguably the most commonly used interest rate curve. This rate curve roughly reflects the default risk of a commercial bank with a credit rating

of A1/A+.

EXAMPLE: Swap spread

The two-year fixed-for-floating swap rate is 2.02% and the two-year U.S. Treasury bond is yielding 1.61%. What is the swap spread?

Answer:

$$\text{swap spread} = (\text{swap rate}) - (\text{T-bond yield})$$
$$= 2.02\% - 1.61\% = 0.41\% \text{ or } 41 \text{ bps}$$

LOS 25.g: Describe short-term interest rate spreads used to gauge economy-wide credit risk and liquidity risk.

I-SPREAD

The **I-spread**, or interpolated spread, for a credit-risky bond is the amount by which the yield on the risky bond exceeds the swap rate for the same maturity. In a case where the swap rate for a specific maturity is not available, the missing swap rate can be estimated from the swap rate curve using linear interpolation.

EXAMPLE: I-spread

6% Zinni, Inc., bonds are currently yielding 2.35% and mature in 1.6 years. From the provided swap curve, compute the I-spread.

Swap curve:

Tenor	Swap Rate
0.5	1.00%
1	1.25%
1.5	1.35%
2	1.50%

Answer:

Linear interpolation:

First, recognize that 1.6 years falls in the 1.5-to-2-year interval.

Interpolated rate = rate for lower bound + (# of years for interpolated rate − # of years for lower bound)(higher bound rate − lower bound rate) / (# of years for upper bound − # of years for lower bound)

1.6-year swap rate =

$$\text{1.5-year swap rate} + \frac{0.10(\text{2-year swap rate} - \text{1.5-year swap rate})}{0.50}$$

$$= 1.35 + \frac{0.10(1.50 - 1.35)}{0.50} = 1.38\%$$

I-spread = yield on the bond − swap rate = 2.35% − 1.38% = 0.97% or 97 bps

While a bond's yield reflects time value as well as compensation for credit and liquidity risk, I-spread only reflects compensation for credit and liquidity risks. The higher the I-spread, the higher the compensation for liquidity and credit risk.

THE *Z*-SPREAD

The **Z-spread** is the spread that, when added to each spot rate on the default-free spot curve, makes the present value of a bond's cash flows equal to the bond's market price. Therefore, the *Z*-spread is a spread over the entire spot rate curve.

For example, suppose the one-year spot rate is 4% and the two-year spot rate is 5%. The market price of a two-year bond with annual coupon payments of 8% is \$104.12. The *Z*-spread is the spread that balances the following equality:

$$\$104.12 = \frac{\$8}{(1 + 0.04 + Z)} + \frac{\$108}{(1 + 0.05 + Z)^2}$$

In this case, the *Z*-spread is 0.008, or 80 basis points. (Plug Z = 0.008 into the right-hand-side of the equation just listed to reassure yourself that the present value of the bond's cash flows equals \$104.12).

The term *zero volatility* in the *Z*-spread refers to the assumption of zero interest rate volatility. *Z*-spread is not appropriate to use to value bonds with embedded options; without any interest rate volatility options are meaningless. If we ignore the embedded options for a bond and estimate the *Z*-spread, the estimated *Z*-spread will include the cost of the embedded option (i.e., it will reflect compensation for option risk as well as compensation for credit and liquidity risk).

EXAMPLE: Computing the price of an option-free risky bond using *Z*-spread

A three-year, 5% annual-pay ABC, Inc., bond trades at a *Z*-spread of 100 bps over the benchmark spot rate curve.

The benchmark one-year spot rate, one-year forward rate in one year and one-year forward rate in year 2 are 3%, 5.051%, and 7.198%, respectively.

Compute the bond's price.

Answer:

First derive the spot rates:

$S_1 = 3\%$ (given)

$(1 + S_2)^2 = (1 + S_1)[1 + f(1,1)] = (1.03)(1.05051) \rightarrow S_2 = 4.02\%$

$(1 + S_3)^3 = (1 + S_1)[1 + f(1,1)]\,[1 + f(2,1)] = (1.03)(1.05051)(1.07198) \rightarrow$
$S_3 = 5.07\%$

value (with *Z*-spread) =

$$\frac{5}{(1.03 + 0.01)} + \frac{5}{(1.0402 + 0.01)^2} + \frac{105}{(1.0507 + 0.01)^3} = 97.33$$

TED Spread

The "TED" in "**TED spread**" is an acronym that combines the "T" in "*T*-bill" with "*ED*" (the ticker symbol for the Eurodollar futures contract).

Conceptually, the TED spread is the amount by which the MRR exceeds the interest rate on short-term U.S. government debt of the same maturity.

For example, if three-month MRR is 0.33% and the three-month T-bill rate is 0.03%, then:

TED spread = (3-month MRR) – (3-month T-bill rate) = 0.33% – 0.03% = 0.30% or 30 bps.

Because T-bills are considered to be risk free while MRR reflects the risk of lending to commercial banks, the TED spread is seen as an indication of the credit and liquidity risk in the banking sector. A rising TED spread indicates that market participants believe banks are increasingly likely to default on loans and that risk-free T-bills are becoming more valuable in comparison. The TED spread captures the risk in the banking system more accurately than does the 10-year swap spread.

MRR-OIS Spread

OIS stands for overnight indexed swap and represents interest rate on unsecured overnight lending between banks. The OIS rate roughly reflects the federal funds rate and includes minimal counterparty credit risk.

The **MRR-OIS spread** (formerly the LIBOR-OIS spread) is the amount by which the MRR (which includes some credit risk) exceeds the OIS rate (which includes only minimal credit risk) and also indicates the level of credit and liquidity risk in the banking system.

MODULE QUIZ 25.4

1. The swap spread for a default-free bond is *least likely* to reflect the bond's:
   A. mispricing in the market.
   B. illiquidity.
   C. time value.
2. Which of the following statements about the Z-spread is *most accurate*? The Z-spread is the:
   A. difference between the yield to maturity of a bond and the linearly interpolated swap rate.
   B. spread over the Treasury spot curve that a bond would trade at if it had zero embedded options.
   C. spread over the Treasury spot curve required to match the value of a bond to its current market price.
3. The TED spread is calculated as the difference between:
   A. the three-month MRR and the three-month T-bill rate.
   B. MRR and the overnight indexed swap rate.
   C. the three-month T-bill rate and the overnight indexed swap rate.

MODULE 25.5: TERM STRUCTURE THEORY

Video covering this content is available online.

LOS 25.h: Explain traditional theories of the term structure of interest rates and describe the implications of each theory for forward rates and the shape of the yield curve.

We'll explain each of the theories of the **term structure of interest rates**, paying particular attention to the implications of each theory for the shape of the yield curve and the interpretation of forward rates.

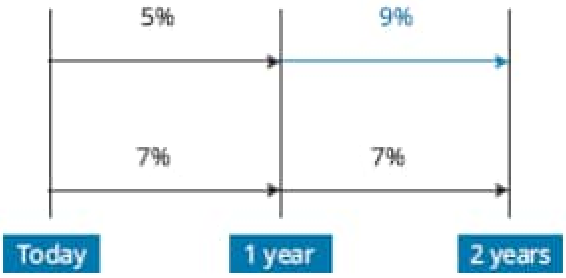
Unbiased Expectations Theory

Under the **unbiased expectations theory** or the **pure expectations theory**, we hypothesize that it is investors' expectations that determine the shape of the interest rate term structure.

Specifically, this theory suggests that forward rates are solely a function of expected future spot rates, and that every maturity strategy has the same expected return over a given investment horizon. In other words, long-term interest rates equal the mean of future *expected* short-term rates. This implies that an investor should earn the same return by investing in a five-year bond or by investing in a three-year bond and then a two-year bond after the three-year bond matures. Similarly, an investor with a three-year investment horizon would be indifferent between investing in a three-year bond or in a five-year bond that will be sold two years prior to maturity. The underlying principle behind the pure expectations theory is risk neutrality: Investors don't demand a risk premium for maturity strategies that differ from their investment horizon.

For example, suppose the one-year spot rate is 5% and the two-year spot rate is 7%. Under the unbiased expectations theory, the one-year forward rate in one year must be 9% because investing for two years at 7% yields approximately the same annual return as investing for the first year at 5% and the second year at 9%. In other words, the two-year rate of 7% is the average of the expected future one-year rates of 5% and 9%. This is shown in Figure 25.3.

Figure 25.3: Spot and Future Rates



Notice that in this example, because short-term rates are expected to rise (from 5% to 9%), the yield curve will be upward sloping.

Therefore, the implications for the shape of the yield curve under the pure expectations theory are:

- If the yield curve is upward sloping, short-term rates are expected to rise.
- If the curve is downward sloping, short-term rates are expected to fall.
- A flat yield curve implies that the market expects short-term rates to remain constant.

Local Expectations Theory

The **local expectations theory** is similar to the unbiased expectations theory with one major difference: the local expectations theory preserves the risk-neutrality assumption only for short holding periods. In other words, over longer periods, risk premiums should exist. This implies that over short time periods, every bond (even long-maturity risky bonds) should earn the risk-free rate.

The local expectations theory can be shown not to hold because the short-holding-period returns of long-maturity bonds can be shown to be higher than short-holding-period returns on short-maturity bonds due to liquidity premiums and hedging concerns.

Liquidity Preference Theory

The **liquidity preference theory** of the term structure addresses the shortcomings of the pure expectations theory by proposing that forward rates reflect investors' expectations of future spot rates, plus a liquidity premium to compensate investors for exposure to interest rate risk. Furthermore, the theory suggests that this liquidity premium is positively related to maturity: a 25-year bond should have a larger liquidity premium than a five-year bond.

Thus, the liquidity preference theory states that forward rates are *biased* estimates of the market's expectation of future rates because they include a liquidity premium. Therefore, a positive-sloping yield curve may indicate that either: (1) the market expects future interest rates to rise or (2) rates are expected to remain constant (or even fall), but the addition of the liquidity premium results in a positive slope. A downward-sloping yield curve indicates steeply falling short-term rates according to the liquidity preference theory.

The size of the liquidity premiums need not be constant over time. They may be larger during periods of greater economic uncertainty when risk aversion among investors is higher.

Segmented Markets Theory

Under the **segmented markets theory**, yields are not determined by liquidity premiums and expected spot rates. Rather, the shape of the yield curve is determined by the preferences of borrowers and lenders, which drives the balance between supply of and demand for loans of different maturities. This is called the segmented markets theory because the theory suggests that the yield at each maturity is determined independently of the yields at other maturities; we can think of each maturity to be essentially unrelated to other maturities.

The segmented markets theory supposes that various market participants only deal in securities of a particular maturity because they are prevented from operating at different maturities. For example, pension plans and insurance companies primarily purchase long-maturity bonds for asset-liability matching reasons and are unlikely to participate in the market for short-term funds.

Preferred Habitat Theory

The **preferred habitat theory** also proposes that forward rates represent expected future spot rates plus a premium, but it does not support the view that this premium

is directly related to maturity.

Instead, the preferred habitat theory suggests that the existence of an imbalance between the supply and demand for funds in a given maturity range will induce lenders and borrowers to shift from their preferred habitats (maturity range) to one that has the opposite imbalance. However, to entice investors to do so, the investors must be offered an incentive to compensate for the exposure to price and/or reinvestment rate risk in the less-than-preferred habitat. Borrowers require cost savings (i.e., lower yields) and lenders require a yield premium (i.e., higher yields) to move out of their preferred habitats.

Under this theory, premiums are related to supply and demand for funds at various maturities. Unlike the liquidity preference theory, under the preferred habitat theory a 10-year bond might have a higher or lower risk premium than the 25-year bond. It also means that the preferred habitat theory can be used to explain almost any yield curve shape.

MODULE QUIZ 25.5

1. Which of the following statements regarding the traditional theories of the term structure of interest rates is *most accurate*?
   A. The segmented markets theory proposes that market participants have strong preferences for specific maturities.
   B. The liquidity preference theory hypothesizes that the yield curve must always be upward sloping.
   C. The preferred habitat theory states that yields at different maturities are determined independently of each other.

MODULE 25.6: YIELD CURVE RISKS AND ECONOMIC FACTORS

Video covering this content is available online.

LOS 25.i: Explain how a bond's exposure to each of the factors driving the yield curve can be measured and how these exposures can be used to manage yield curve risks.

MANAGING YIELD CURVE RISKS

Yield curve risk refers to risk to the value of a bond portfolio due to unexpected changes in the yield curve.

To counter yield curve risk, we first identify our portfolio's sensitivity to yield curve changes using one or more measures. Yield curve sensitivity can be generally measured by **effective duration**, or more precisely using **key rate duration**, or a three-factor model that decomposes changes in the yield curve into changes in **level**, **steepness**, and **curvature**.

Effective Duration

Effective duration measures price sensitivity to small *parallel* shifts in the yield curve. It is important to note that effective duration is not an accurate measure of interest rate sensitivity to *non-parallel* shifts in the yield curve like those described

by **shaping risk**. Shaping risk refers to changes in portfolio value due to changes in the *shape* of the benchmark yield curve. (Note, however, that parallel shifts explain more than 75% of the variation in bond portfolio returns.)

Key Rate Duration

A more precise method used to quantify bond price sensitivity to interest rates is key rate duration. Compared to effective duration, key rate duration is superior for measuring the impact of nonparallel yield curve shifts.

Key rate duration is the sensitivity of the value of a security (or a bond portfolio) to changes in a single par rate, holding all other par rates constant. In other words, key rate duration isolates price sensitivity to a change in the yield at a particular maturity only.

Numerically, key rate duration is defined as the approximate percentage change in the value of a bond portfolio in response to a 100 basis point change in the corresponding key par rate, holding all other par rates constant. Conceptually, we could determine the key rate duration for the five-year segment of the yield curve by changing only the five-year par rate and observing the change in value of the portfolio. Keep in mind that every security or portfolio has a set of key rate durations—one for each key rate.

For example, a bond portfolio has interest rate risk exposure to only three maturity points on the par rate curve: the 1-year, 5-year, and 25-year maturities, with key rate durations represented by $D_1 = 0.7$, $D_5 = 3.5$, and $D_{25} = 9.5$, respectively. The sum total of these key rate durations is the effective duration of the portfolio.

The model for yield curve risk using these key rate durations would be:

$$\frac{\Delta P}{P} \approx -D_1\Delta r_1 - D_5\Delta r_5 - D_{25}\Delta r_{25}$$

$$\frac{\Delta P}{P} \approx -(0.7)\Delta r_1 - (3.5)\Delta r_5 - (9.5)\Delta r_{25}$$

Sensitivity to Parallel, Steepness, and Curvature Movements

An alternative to decomposing yield curve risk into sensitivity to changes at various maturities (key rate duration) is to decompose the risk into sensitivity to the following three categories of yield curve movements:

- **Level** (Δx_L). A parallel increase or decrease of interest rates.
- **Steepness** (Δx_S). Long-term interest rates increase while short-term rates decrease.
- **Curvature** (Δx_C). Increasing curvature means short- and long-term interest rates increase while intermediate rates do not change.

It has been found that all yield curve movements can be described using a combination of one or more of these movements.

We can then model the change in the value of our portfolio as follows:

$$\frac{\Delta P}{P} \approx -D_L \Delta x_L - D_S \Delta x_S - D_C \Delta x_C$$

where D_L, D_S, and D_C are respectively the portfolio's sensitivities to changes in the yield curve's level, steepness, and curvature.

For example, for a particular portfolio, yield curve risk can be described as:

$$\frac{\Delta P}{P} \approx -4\Delta x_L - 5\Delta x_S - 3\Delta x_C$$

If the following changes in the yield curve occurred: $\Delta x_L = -0.004$, $\Delta x_S = 0.001$, and $\Delta x_C = 0.002$, then the percentage change in portfolio value could be calculated as:

$$\frac{\Delta P}{P} \approx -4(-0.004) - 5(0.001) - 3(0.002) = 0.005$$

This predicts a +0.5% increase in the portfolio value resulting from the yield curve movements.

LOS 25.j: Explain the maturity structure of yield volatilities and their effect on price volatility.

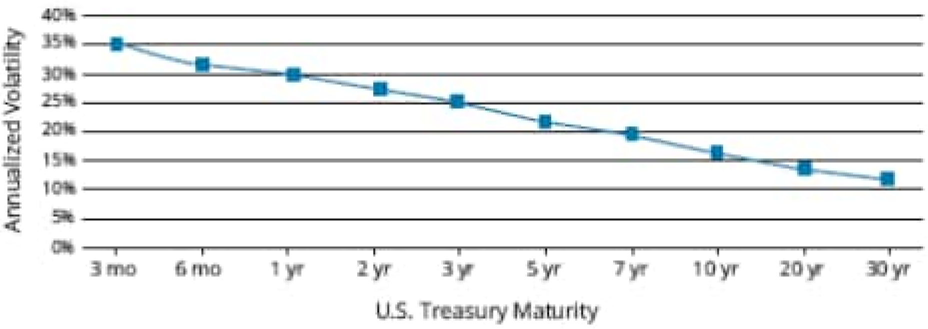
MATURITY STRUCTURE OF YIELD CURVE VOLATILITIES

Interest rate volatility is a key concern for bond managers because interest rate volatility drives price volatility in a fixed income portfolio. Interest rate volatility becomes particularly important when securities have embedded options, which are especially sensitive to volatility.

The **term structure of interest rate volatility** is the graph of yield volatility versus maturity.

Figure 25.4 shows a typical term structure of interest rate volatility. Note that, as shown here, short-term interest rates are generally more volatile than are long-term rates.

Figure 25.4: Historical Volatility Term Structure: U.S. Treasuries, August 2005–December 2007



Volatility at the long-maturity end is thought to be associated with uncertainty regarding the real economy and inflation, while volatility at the short-maturity end reflects risks regarding monetary policy.

Interest rate volatility at time *t* for a security with maturity of *T* is denoted as σ(t,T). This variable measures the annualized standard deviation of the change in bond yield.

LOS 25.k: Explain how key economic factors are used to establish a view on benchmark rates, spreads, and yield curve changes.

Macroeconomic factors that affect bond yields include inflation forecasts, GDP growth, and monetary policy. Two-thirds of the *variation* in short- and intermediate-term yields is explained by monetary policy, and the remaining is explained by the other factors. In contrast, inflation explains two-thirds of the variation in long-term yields, with the remaining mostly explained by monetary policy.

Bond risk premium is the excess return (over the one-year risk-free rate) earned by investors for investing in long-term government bonds. Bond risk premium (also known as the *term premium* or *duration premium*) is a forward-looking expectation. For example, suppose that the expected one-year holding period return on a 5-year government bond is 4%, while the one-year risk-free rate is 1%. The 5-year bond risk premium in this example would be 4% – 1% = 3%.

Monetary policy influences the shape of the yield curve. During economic expansions, to combat rising inflation, central banks may raise short-term rates, leading to a **bearish flattening** of the yield curve. During recessionary times, central banks may reduce short-term rates, leading to a **bullish steepening**. The U.S. Federal Reserve has dramatically expanded its balance sheet in recent years, purchasing U.S. Treasuries and mortgage-backed and municipal securities, reducing long-term bond yields.

Other factors affecting bond prices include the following:

- *Fiscal policy*. Expansionary (restrictive) fiscal policies increase (decrease) yields.
- *Maturity structure*. The government's choice of maturity when issuing new securities affects the supply (and yield) of bonds in those maturity segments. An increase in offerings in a specific segment of the market increases the supply and increases the yield in that segment (the market segmentation theory).
- *Investor demand*. Domestic and foreign investor demand and preferences for specific maturity segments affect the yield in that segment. For example, pension plans and insurance companies may prefer longer maturity bonds, driving down longer-term yields. During periods of market turmoil, a **flight to safety** may reduce long-term government bond yields, resulting in a **bullish flattening** of the yield curve.

Investor Actions

In expectation of a rise (fall) in rates, investors will lower (extend) the duration of their bond portfolios. Expectations of a steepening of the yield curve may lead to investors going long short-term bonds and short longer-term bonds. Trades may be designed to be duration-neutral so that a change in the *level* of interest rates does not affect the value of the portfolio. Investors with long-only mandates may rotate between a **bullet portfolio** (a portfolio concentrated in a single maturity) and a

barbell portfolio (a portfolio with short and long maturities). An investor that expects a bullish flattening of the yield curve may rotate out of a bullet portfolio and into a barbell portfolio.

MODULE QUIZ 25.6

1. The *least appropriate* measure to use to identify and manage "shaping risk" is a portfolio's:
   A. effective duration.
   B. key rate durations.
   C. sensitivities to level, steepness, and curvature factors.
2. Regarding the volatility term structure, research indicates that volatility in short-term rates is *most strongly* linked to uncertainty regarding:
   A. the real economy.
   B. monetary policy.
   C. inflation.
3. Restrictive monetary policy is *most likely* to be associated with:
   A. bearish flattening.
   B. bullish steepening.
   C. bullish flattening.

KEY CONCEPTS

LOS 25.a

The spot rate for a particular maturity is equal to a geometric average of the one-period spot rate and a series of one-period forward rates.

When the spot curve is flat, forward rates will equal spot rates and yields. When the spot curve is upward sloping (downward sloping), forward rate curves will be above (below) the spot curve and the yield for a maturity of *T* will be less than (greater than) the spot rate S_T.

The forward *pricing* model values forward contracts by using an arbitrage-free framework that equates buying a zero-coupon bond to entering into a forward contract to buy a zero-coupon bond in the future that matures at the same time:

$$P_{(j+k)} = P_j F_{(j,k)}$$

The forward *rate* model tells us that the investors will be indifferent between buying a long-maturity zero-coupon bond versus buying a shorter-maturity zero-coupon bond and reinvesting the principal at the locked in forward rate *f*(j,k).

$$[1 + S_{(j+k)}]^{(j+k)} = (1 + S_j)^j \, [1 + f(j,k)]^k$$

LOS 25.b

By using a process called bootstrapping, spot rates (i.e., zero-coupon rates) can be derived from the par curve iteratively—one spot rate at a time.

LOS 25.c

If spot rates evolve as predicted by forward rates, bonds of all maturities will realize a one-period return equal to the one-period spot rate and the forward price will remain unchanged.

Active bond portfolio management is built on the presumption that the current forward curve may not accurately predict future spot rates. Managers attempt to outperform the market by making predictions about how spot rates will change relative to the rates suggested by forward rate curves.

If an investor believes that future spot rates will be lower than corresponding forward rates, then the investor will purchase bonds (at a presumably attractive price) because the market appears to be discounting future cash flows at "too high" of a discount rate.

LOS 25.d

When the yield curve is upward sloping, bond managers may use the strategy of "rolling down the yield curve" to chase above-market returns. By holding long-maturity rather than short-maturity bonds, the manager earns an excess return as the bond "rolls down the yield curve" (i.e., approaches maturity and increases in price). As long as the yield curve remains upward sloping and the spot rates continue to be lower than previously implied by their corresponding forward rates, this strategy will add to the return of a bond portfolio.

LOS 25.e

The swap rate curve provides a benchmark measure of interest rates. It is similar to the yield curve except that the rates used represent the interest rates of the fixed-rate leg in an interest rate swap.

Market participants prefer the swap rate curve as a benchmark interest rate curve rather than a government bond yield curve for the following reasons:

1. Swap rates reflect the credit risk of commercial banks rather than that of governments.
2. The swap market is not regulated by any government.
3. The swap curve typically has yield quotes at many maturities.

LOS 25.f

We define swap spread as the additional interest rate paid by the fixed-rate payer of an interest rate swap over the rate of the "on-the-run" government bond of the same maturity.

swap spread = (swap rate) − (Treasury bond yield)

Investors use the swap spread to separate the time value portion of a bond's yield from the risk premiums for credit and liquidity risk. The higher the swap spread, the higher the compensation for liquidity and credit risk.

For a default-free bond, the swap spread provides an indication of (1) the bond's liquidity and/or (2) possible mispricing.

LOS 25.g

The *Z*-spread is the spread that when added to each spot rate on the yield curve makes the present value of a bond's cash flows equal to the bond's market price. The *Z* refers to zero volatility—a reference to the fact that the *Z*-spread assumes interest rate volatility is zero. *Z*-spread is not appropriate to use to value bonds with embedded options.

TED spreads

TED is an acronym that combines T-bill and ED *("ED" is the ticker symbol for the Eurodollar futures contract)*.

TED spread = (MRR) − (T-bill rate)

The TED spread is used as an indication of the perceived credit and liquidity risk in the economy. Because it captures the spread between rates paid by large banks and those paid by the U.S. Treasury, the TED spread reflects the default risk in the banking sector.

MRR-OIS spread

The MRR-OIS spread is the amount by which the MRR (which includes some credit risk) exceeds the overnight indexed swap (OIS) rate (which includes only minimal credit risk).

LOS 25.h

There are several traditional theories that attempt to explain the term structure of interest rates:

Unbiased expectations theory—Forward rates are an unbiased predictor of future spot rates. Also known as the *pure expectations theory*.

Local expectations theory—Bond maturity does not influence returns for short holding periods.

Liquidity preference theory—Investors demand a liquidity premium that is positively related to a bond's maturity.

Segmented markets theory—The shape of the yield curve is the result of the interactions of supply and demand for funds in different market (i.e., maturity) segments.

Preferred habitat theory—Similar to the segmented markets theory, but recognizes that market participants will deviate from their preferred maturity habitat if compensated adequately.

LOS 25.i

We can measure a bond's exposures to the factors driving the yield curve in a number of ways:

1. **Effective duration**—Measures the sensitivity of a bond's price to *parallel* shifts in the benchmark yield curve.
2. **Key rate duration**—Measures bond price sensitivity to a change in a specific spot rate keeping everything else constant.
3. **Sensitivity to parallel, steepness, and curvature movements**—Measures sensitivity to three distinct categories of changes in the shape of the benchmark yield curve.

LOS 25.j

The maturity structure of yield volatilities indicates the level of yield volatilities at different maturities. This term structure thus provides an indication of yield curve

risk. The volatility term structure usually indicates that short-term rates (which are linked to uncertainty over monetary policy) are more volatile than long-term rates (which are driven by uncertainty related to the real economy and inflation). Fixed income instruments with embedded options can be especially sensitive to interest rate volatility.

LOS 25.k

Inflation forecasts, GDP growth, and monetary policy affect bond yields. Bond risk premium is the excess return (over the one-year risk-free rate) from investing in longer-term government bonds. Additionally, fiscal policy, maturity structure, and investor demand all affect yields.

In expectation of a rise (fall) in rates, investors will lower (extend) the duration of their bond portfolios. An investor will rotate out of a bullet portfolio and into a barbell portfolio in expectation of a bullish flattening of the yield curve.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 25.1

1. **B** When the yield curve is *upward* sloping, the forward curves will lie above the spot curve. The opposite is true when the yield curve is downward sloping. (LOS 25.a)

2. **B** The forward *pricing* model values forward contracts by using an arbitrage argument that equates buying a zero-coupon bond to entering into a forward contract to buy a zero-coupon bond that matures at the same time:

$$P_{(j+k)} = P_j F_{(j,k)}$$

The forward *rate* model tells us that the forward rate *f*(j,k) should make investors indifferent between buying a long-maturity zero-coupon bond versus buying a shorter-maturity zero-coupon bond and reinvesting the principal. (LOS 25.a)

Module Quiz 25.2

1. **B** If an investor believes that future spot rates will be lower than the current forward rates, then the investor will perceive an opportunity to purchase bonds at an attractive price, as the market is discounting future cash flows at "too high" a discount rate. The bonds are thus undervalued in the market. (LOS 25.c)

2. **C** Fixed income managers will earn an extra return through rolling down the yield curve if the spot rate curve is upward sloping and remains unchanged over time. (LOS 25.d)

Module Quiz 25.3

1. **C** The swap market is not controlled by governments, which makes swap rates more comparable across different countries. The swap rate is the interest rate for the fixed-rate leg of an interest rate swap. *Wholesale* banks frequently use the swap curve to value their assets and liabilities, while *retail* banks with

little exposure to the swap market are more likely to use the government spot curve as their benchmark. (LOS 25.e)

Module Quiz 25.4

1. **C** The swap spread of a default free bond should provide an indication of the bond's illiquidity—or, alternatively, that the bond is mispriced. Time value is reflected in the government bond yield curve; the swap spread is an additional amount of interest above this benchmark. (LOS 25.f)

2. **C** The *Z*-spread is the constant spread that must be added to the default-free spot curve to match the valuation of a risky bond to its market price. A higher *Z*-spread implies a riskier bond. (LOS 25.g)

3. **A** The TED spread (from *T*-bill and *E*uro*d*ollar) is computed as the difference between the three-month MRR and the three-month T-bill rate. The MRR-OIS spread is the difference between MRR and the overnight indexed swap rate (OIS) rates. (LOS 25.g)

Module Quiz 25.5

1. **A** The segmented markets theory (and the preferred habitat theory) propose that borrowers and lenders have strong preferences for particular maturities. The liquidity preference theory argues that there are liquidity premiums that increase with maturity; however, the liquidity preference theory does not preclude the existence of other factors that could lead to an overall downward-sloping yield curve. The segmented markets theory—not the preferred habitat theory—proposes that yields at different maturities are determined independently of each other. (LOS 25.h)

Module Quiz 25.6

1. **A** Effective duration is an inappropriate measure for identifying and managing shaping risk. Shaping risk refers to risk to portfolio value from changes in the shape of the benchmark yield curve. Effective duration can be used to accurately measure the risk associated with parallel yield curve changes but is not appropriate for measuring the risk from other changes in the yield curve. (LOS 25.i)

2. **B** It is believed that short-term volatility reflects uncertainty regarding monetary policy while long-term volatility is most closely associated with uncertainty regarding the real economy and inflation. Short-term rates in the volatility term structure tend to be more volatile than long-term rates. (LOS 25.j)

3. **A** During economic expansions, restrictive monetary policy to control inflation leads to an increase in short-term rates and a bearish flattening of the yield curve. (LOS 25.k)

READING 26

THE ARBITRAGE-FREE VALUATION FRAMEWORK

EXAM FOCUS

This topic review discusses valuation of fixed-income securities using spot rates as well as using the backward induction methodology in a binomial interest rate tree framework. Understand how embedded options impact the suitability of the binomial model or the Monte Carlo simulation method.

Video covering this content is available online.

MODULE 26.1: BINOMIAL TREES, PART 1

LOS 26.a: Explain what is meant by arbitrage-free valuation of a fixed-income instrument.

The arbitrage-free valuation framework is used extensively in the pricing of securities. The basic principle of the "law of one price" in freely functioning markets drives this analytical framework.

Arbitrage-free valuation methods value securities such that no market participant can earn an arbitrage profit in a trade involving that security. An arbitrage transaction involves no initial cash outlay but a positive riskless profit (cash flow) at some point in the future.

There are two types of arbitrage opportunities: **value additivity** (when the value of whole differs from the sum of the values of parts) and **dominance** (when one asset trades at a lower price than another asset with identical characteristics).

If the principle of value additivity does not hold, arbitrage profits can be earned by **stripping** or **reconstitution**. A five-year, 5% Treasury bond should be worth the same as a portfolio of its coupon and principal strips. If the portfolio of strips is trading for less than an intact bond, one can purchase the strips, combine them (reconstituting), and sell them as a bond. Similarly, if the bond is worth less than its component parts, one could purchase the bond, break it into a portfolio of strips (stripping), and sell those components.

EXAMPLE: Arbitrage opportunities

The following information has been collected:

Security	Current Price	Payoff in 1 Year
A	$99	$100
B	$990	$1,010
C	$100	$102
D	$100	$103

Securities A and B are identical in every respect other than as noted. Similarly, securities C and D are identical in every other respect.

Demonstrate the exploitation of any arbitrage opportunities.

Answer:

1. Arbitrage due to violation of the value additivity principle:

	Cash Flow	
	t = 0	t = 1
Short 10 units of Security A	+$990	–$1,000
Long 1 unit of Security B	–$990	+$1,010
Net cash flow	-0-	+$10

2. Arbitrage due to the occurrence of dominance:

	Cash Flow	
	t = 0	t = 1
Short 1 unit of Security C	+\$100	–\$102
Long 1 unit of Security D	–\$100	+\$103
Net cash flow	-0-	+\$1

LOS 26.b: Calculate the arbitrage-free value of an option-free, fixed-rate coupon bond.

Arbitrage-free valuation of a fixed-rate, option-free bond entails discounting each of the bond's future cash flows (i.e., each coupon payment and the par value at maturity) using the corresponding spot rate.

EXAMPLE: Arbitrage-free valuation

Sam Givens, a fixed income analyst at GBO Bank, has been asked to value a three-year, 3% annual pay, €100 par bond with the same liquidity and risk as the benchmark. What is the value of the bond using the spot rates provided in the following?

€ Benchmark Spot Rate Curve:

Year	Spot Rate
1	3.00%
2	3.25%
3	3.50%

Answer:

$$\text{Bond value} = \frac{3}{(1.03)} + \frac{3}{(1.0325)^2} + \frac{103}{(1.0350)^3} = €98.63$$

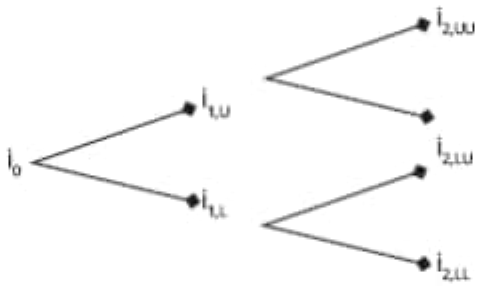
While we can value option-free bonds with a simple spot rate curve, for bonds with embedded options, changes in future rates will affect the probability of the option being exercised and the underlying future cash flows. Thus, a model that allows both rates and the underlying cash flows to vary should be used to value bonds with embedded options. One such model, the binomial interest rate tree framework, is discussed next.

LOS 26.c: Describe a binomial interest rate tree framework.

The **binomial interest rate tree** framework assumes that interest rates have an equal probability of taking one of two possible values in the next period (hence the term binomial).

Over multiple periods, the set of possible interest rate paths that are used to value bonds with a binomial model is called a binomial interest rate tree. The diagram in Figure 26.1 depicts a binomial interest rate tree.

Figure 26.1: Binomial Interest Rate Tree



The starting point i_0 in this tree is the current 1-period spot rate and is called the root of the tree. The future nodes are indicated with the bold dots (♦). A node is a point in time when interest rates can take one of two possible paths, an upper path, *U*, or a lower path, *L*. Now, consider the node on the right side of the diagram where the interest rate $i_{2,LU}$ appears. This is the rate that will occur if the initial rate, i_0, follows the lower path from node 0 to node 1 to become $i_{1,L}$, then follows the upper of the two possible paths to node 2, where it takes on the value $i_{2,LU}$. At the risk of stating the obvious, the upper path from a given node leads to a higher rate than the lower path. In fact,

$$i_{2,LU} = i_{2,LL}e^{2\sigma}$$

where:

$e \approx 2.7183$ (i.e., the base of natural log)

σ = standard deviation of interest rates (i.e., the interest rate volatility used in the model)

Note that an upward move followed by a downward move, or a down-then-up move, produces the same result: $i_{2,LU} = i_{2,UL}$ (the math term for this is lattice).

The interest rates at each node in this interest rate tree are one-period forward rates corresponding to the nodal period. Each forward rate is related to (i.e., is a multiple of) the other forward rates in the same nodal period. Adjacent forward rates (at the same period) are two standard deviations apart. For the first period, there are two forward rates and hence:

$$i_{1,U} = i_{1,L}e^{2\sigma}$$

PROFESSOR'S NOTE

Consistent with the curriculum, I use the term "forward rates" to denote the "future one-period expected spot rates," even though they are not the same.

Beyond the first nodal period, non-adjacent forward rates are a multiple of the two standard deviations (depending on how separated the forward rates are).

For example,

$$i_{2,UU} = i_{2,LL}e^{4\sigma}$$

The relationship among the set of rates associated with each individual nodal period is a function of the interest rate volatility assumed to generate the tree. Volatility estimates can be based on historical data or can be implied volatility derived from interest rate derivatives.

The binomial interest rate tree framework is a lognormal random walk model with two properties: (1) higher volatility at higher rates and (2) non-negative interest rates.

LOS 26.e: Describe the backward induction valuation methodology and calculate the value of a fixed-income instrument given its cash flow at each node.

VALUING AN OPTION-FREE BOND WITH THE BINOMIAL MODEL

Backward induction refers to the process of valuing a bond using a binomial interest rate tree. The term "backward" is used because in order to determine the value of a bond today at Node 0, you need to know the values that the bond can take at the Year 1 node. But to determine the values of the bond at the Year 1 nodes, you need to know the possible values of the bond at the Year 2 nodes. Thus, for a bond that has *N* compounding periods, the current value of the bond is determined by computing the bond's possible values at Period *N* and working backwards to Node 0.

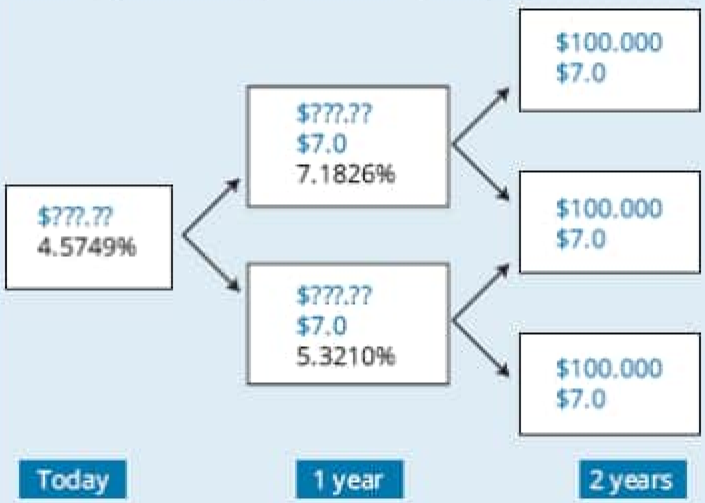
Because the probabilities of an up move and a down move are both 50%, the value of a bond at a given node in a binomial tree is the average of the present values of the two possible values from the next period. The appropriate discount rate is the forward rate associated with the node.

The following example should make this all clear.

EXAMPLE: Valuing an option-free bond with the binomial model

A 7% annual coupon bond has two years to maturity. The interest rate tree is shown in the following figure. Fill in the tree and calculate the value of the bond today.

Valuing a 2-Year, 7.0% Coupon, Option-Free Bond



Answer:

Consider the value of the bond at the *upper* node for Period 1 ($V_{1,U}$):

$$V_{1,U} = \frac{1}{2} \times \left[\frac{\$100 + \$7}{1.071826} + \frac{\$100 + \$7}{1.071826}\right] = \$99.830$$

Similarly, the value of the bond at the *lower* node for Period 1 ($V_{1,L}$) is:

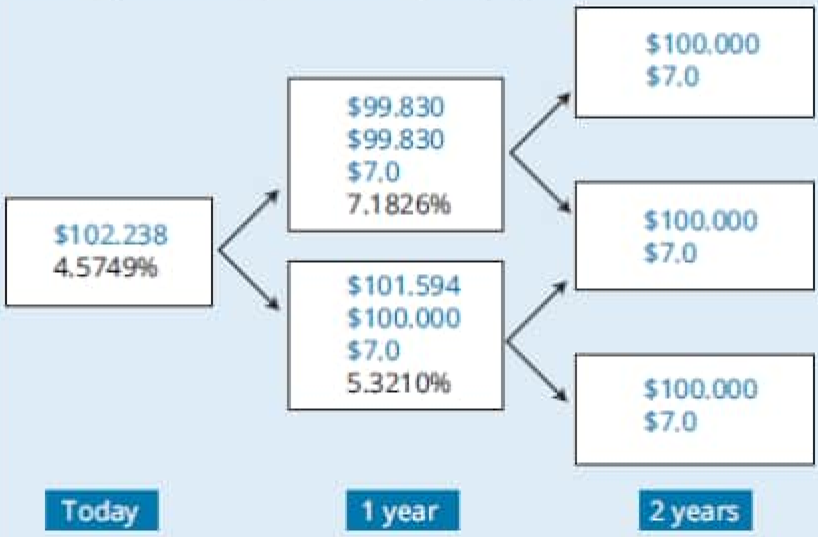
$$V_{1,L} = \frac{1}{2} \times \left[\frac{\$100 + \$7}{1.053210} + \frac{\$100 + \$7}{1.053210}\right] = \$101.594$$

Now calculate V_0, the current value of the bond at Node 0.

$$V_0 = \frac{1}{2} \times \left[\frac{\$99.830 + \$7}{1.045749} + \frac{\$101.594 + \$7}{1.045749}\right] = \$102.999$$

The completed binomial tree is shown as follows:

Valuing a 2-Year, 7.0% Coupon, Option-Free Bond



MODULE QUIZ 26.1

Use the following information to answer Questions 1 through 5.

Dan Green, CFA, is currently working for FIData, a company specializing in the provision of price data for fixed-income instruments.

Green heads a team that inputs raw data into an instructional area on FIData's website. FIData uses these pages to provide hypothetical bond information along with a description of how to read and interpret the information.

One of FIData's customers has questioned whether some of the data used to demonstrate pricing concepts is correct. The customer's email stated, "The prices of the three bonds used are not consistent with each other and hence may not be accurate. If they were, I would be able to make a significant arbitrage profit (i.e., I could secure the current risk-free rate of return with zero net investment)." Green thinks the customer is misinformed regarding arbitrage gains, but wants to check the data anyway.

The relevant data for three hypothetical risk-free bonds is shown in Figure 1 given that the benchmark yield curve is flat at 1.50%. (FFPQ is an annual-pay bond.)

Figure 1: Bond Pricing Data

Bond Issue	Maturity	Coupon	Par ($)	Price ($)
FFPQ	2 years	10%	$1,000	1,170.12
DALO	1 year	0%	$1,000	985.22
NKDS	2 years	0%	$11,000	10,677.28

In another area of the company's instructional website, FIData has an explanation of the binomial interest rate tree framework that its analysts use in their valuation process. All of FIData's models populate a binomial interest rate tree assuming interest rates follow a lognormal random walk.

The web page makes two statements regarding the assumptions that underpin the construction and population of such trees:

Assumption 1: The lognormal model ensures that interest rates are non-negative.

Assumption 2: The lognormal model ensures a constant volatility of interest rates at all levels of interest rates.

An example of a standard tree used by FIData is given in Figure 2.

Figure 2: Binomial Interest Rate Tree

Year 0	Year 1
4.5749%	7.1826%
	5.3210%

FIData's website uses rates in Figure 2 to value a two-year, 5% annual-pay coupon bond with a par value of $1,000 using the backward induction method.

1. Green is correct in stating that the customer who sent the email regarding arbitrage gains is misinformed because:
   A. an arbitrage gain always requires a net investment, although this may be small compared to the potential gains.
   B. an arbitrage gain is not constrained by the risk-free rate.
   C. arbitrage gains require a net investment in proportion to returns.
2. Given the three bonds in Figure 1, it is possible to make an arbitrage gain by:
   A. selling 1 FFPQ bond and simultaneously purchasing 10 DALO and 10 NKDS bonds.
   B. selling 10 FFPQ bonds and simultaneously purchasing 1 DALO and 1 NKDS bond.
   C. selling 1 DALO bond and 1 NKDS bond and simultaneously purchasing 10 FFPQ bonds.
3. Which of the following statements regarding the FFPQ bond in Figure 1 is *most likely*?
   A. It is priced above its no arbitrage price.
   B. It is priced at its no arbitrage price.
   C. It is priced below its no arbitrage price.
4. Which of the assumptions regarding the construction of FIData's binomial interest rate trees is *most accurate*?
   A. Assumption 1 only.
   B. Assumption 2 only.
   C. Neither assumption is correct.
5. Using the backward induction method, the value of the 5% annual-pay bond using the interest rate tree given in Figure 2 is *closest* to:
   A. $900.
   B. $945.
   C. $993.

MODULE 26.2: BINOMIAL TREES, PART 2

Video covering this content is available online.

LOS 26.d: Describe the process of calibrating a binomial interest rate tree to match a specific term structure.

The construction of a binomial interest rate tree is a tedious process. In practice, the interest rate tree is usually generated using specialized computer software. The underlying process conforms to three rules:

1. The interest rate tree should generate **arbitrage-free values** for the benchmark security. This means that the value of bonds produced by the interest rate tree must be equal to their market price, which excludes arbitrage opportunities. This requirement is very important because without it, the model will not properly price more complex callable and putable securities, which is the intended purpose of the model.
2. As stated earlier, adjacent forward rates (for the same period) are two standard deviations apart (calculated as $e^{2\sigma}$). Hence, knowing one of the forward rates (out of many) for a particular nodal period allows us to compute the other forward rates for that period in the tree.
3. The middle forward rate (or mid-point in case of even number of rates) in a period is approximately equal to the implied (from the benchmark spot rate curve) one-period forward rate for that period.

EXAMPLE: Binomial interest rate tree

Xi Nguyen, CFA, has collected the following information on the par rate curve, spot rates, and forward rates. Nguyen had asked a colleague, Alok Nath, to generate a binomial interest rate tree consistent with this data and an assumed volatility of 20%. Nath completed a partial interest rate tree shown as follows:

Maturity	Par Rate	Spot Rate
1	3%	3.000%
2	4%	4.020%
3	5%	5.069%

Binomial Tree With σ = 20% (One-Year Forward Rates)

Time 0	Time 1	Time 2
3%	5.7883%	B
	A	C
		D

1. Calculate the forward rate indicated by A.
2. Estimate the forward rate indicated by C.
3. Estimate forward rates B and D.

Answer:

1. Forward rate $i_{1,L}$ is indicated by A and is related to forward rate $i_{1,U}$ given as 5.7883%.

   $i_{1,L} = i_{1,U}\,e^{-2\sigma} = (0.057883)e^{-(2 \times 0.20)} = 0.0388$ or 3.88%

2. Forward rate C is the middle rate for Period 3 and hence the best estimate for that rate is the one-year forward rate in two years $f(2,1)$.

   Using the spot rates, we can bootstrap the forward rate:

   $(1 + S_3)^3 = (1 + S_2)^2(1 + f(2,1)) \rightarrow (1.05069)^3 = (1.0402)^2(1 + f(2,1))$
   $\rightarrow f(2,1) = 7.199\%$

3. The forward rates B and D are related to C as follows (note that $C = i_{2,LU}$):

   $D = i_{2,LL} = i_{2,LU}e^{-2\sigma} = (0.07199)\,e^{-0.40} = 0.0483$ or 4.83%
   $B = i_{2,UU} = i_{2,LU}e^{+2\sigma} = (0.07199)\,e^{+0.40} = 0.1074$ or 10.74%

PROFESSOR'S NOTE

To calculate the value of $e^{-0.40}$ on your TI BA II PLUS calculator, use the following keystrokes: 0.4 [+|–] [2ND] [LN]

LOS 26.f: Compare pricing using the zero-coupon yield curve with pricing using an arbitrage-free binomial lattice.

We have already discussed arbitrage-free valuation of benchmark option-free bonds using the **zero-coupon yield curve** (also known as the spot rate curve). Each known future cash flow is discounted at the underlying spot rate (also known as the zero-coupon yield).

EXAMPLE: Valuation of option-free bond

Samuel Favre is interested in valuing a three-year, 3% annual-pay Treasury bond. He has compiled the following information on Treasury spot rates:

Treasury Spot Rate Curve

Maturity	Spot Rate
1	3.000%
2	4.020%
3	5.069%

Compute the value of the $100 face value option-free bond.

Answer:

$$\text{value of the bond} = \frac{3}{1.03} + \frac{3}{(1.0402)^2} + \frac{103}{(1.05069)^3} = \$94.485$$

For bonds with embedded options, the future cash flows are uncertain as they depend on whether the embedded option will be in the money (and hence exercised). Because the value of the option depends on uncertain future interest rates, the underlying cash flows are also dependent on the same future interest rates.

Hence, to value bonds with embedded options, we have to allow for rates to fluctuate. One way to accomplish this is to use the binomial interest rate tree.

PROFESSOR'S NOTE

The binomial tree approach can value either option-free bonds or bonds with embedded options. Valuation of bonds with embedded options is covered in the next topic review.

EXAMPLE: Valuation of an option-free bond using binomial tree

Samuel Favre is interested in valuing the same three-year, 3% annual-pay Treasury bond. The spot rate curve is as before, but this time Favre wants to use a binomial interest rate tree with the following rates:

One-Period Forward Rate in Year

0	1	2
3%	5.7883%	10.7383%
	3.8800%	7.1981%
		4.8250%

Compute the value of the $100 par option-free bond.

Answer:

$$V_{2,UU} = \frac{103}{(1.107383)} = \$93.01$$

$$V_{2,UL} = \frac{103}{(1.071981)} = \$96.08$$

$$V_{2,LL} = \frac{103}{(1.048250)} = \$98.26$$

$$V_{1,U} = \frac{1}{2} \times \left[\frac{93.01 + 3}{1.057883} + \frac{96.08 + 3}{1.057883}\right] = \$92.21$$

$$V_{1,L} = \frac{1}{2} \times \left[\frac{96.08 + 3}{1.038800} + \frac{98.26 + 3}{1.038800}\right] = \$96.43$$

$$V_0 = \frac{1}{2} \times \left[\frac{92.21 + 3}{1.03} + \frac{96.43 + 3}{1.03}\right] = \$94.485$$

Note that the underlying interest rate tree in this example was calibrated to generate arbitrage-free values consistent with the benchmark spot rate curve and

hence produced the same value for the option-free bond as in the earlier example.

LOS 26.g: Describe pathwise valuation in a binomial interest rate framework and calculate the value of a fixed-income instrument given its cash flows along each path.

Another mathematically identical approach to valuation using the backward induction method in a binomial tree is the **pathwise valuation** approach. For a binomial interest rate tree with *n* periods, there will be $2^{(n-1)}$ unique paths. For example, for a three-period tree, there will be 2^2 or four paths comprising one known spot rate and varying combinations of two unknown forward rates. If we use the label of *S* for the one-period spot rate and *U* and *L* for upper and lower outcomes, respectively, for forward rates in our binomial framework, the four paths would be SUU, SUL, SLU, and SLL.

EXAMPLE: Valuation of option-free bond using pathwise valuation

Samuel Favre wants to value the same three-year, 3% annual-pay Treasury bond. The interest rate tree (shown next) is the same as before but this time, Favre wants to use a pathwise valuation approach.

One-Period Forward Rate in Year

0	1	2
3%	5.7883%	10.7383%
	3.8800%	7.1981%
		4.8250%

Compute the value of the $100 par option-free bond.

Answer:

For a three-year bond, there are four potential interest rate paths. The value of the bond for each path is computed as the sum of the present values of each cash flow discounted at its respective path-specified rate. Pathwise valuation discounts cash flows one year at a time using one-year forward rates (similar to backward induction) rather than spot rates.

Path	Year 1	Year 2	Year 3	Value
1	3%	5.7883%	10.7383%	$91.03
2	3%	5.7883%	7.1981%	$93.85
3	3%	3.8800%	7.1981%	$95.52
4	3%	3.8800%	4.8250%	$97.55
			Average	$94.49

For example, the value of the bond in Path 1 is computed as:

$$\text{value}_1 = \frac{3}{(1.03)} + \frac{3}{(1.03)(1.057883)} + \frac{103}{(1.03)(1.057883)(1.107383)}$$
$$= \$91.03$$

LOS 26.h: Describe a Monte Carlo forward-rate simulation and its application.

Path Dependency

Prepayments on underlying residential mortgages affect the cash flows of a mortgage-backed security. Prepayment risk is similar to call risk in a callable bond. However, unlike call risk, prepayment risk is affected not only by the level of interest rate at a particular point in time, but also by the path rates took to get there.

Consider a mortgage pool that was formed when rates were 6%, then interest rates dropped to 4%, rose to 6%, and then dropped again to 4%. Many homeowners will have refinanced when interest rates dipped the first time. On the second occurrence of 4% interest rates, most homeowners in the pool who were able to refinance would have already taken advantage of the opportunity, leading to lower prepayments than would be observed had 4% interest rates not occurred previously.

An important assumption of the binomial valuation process is that the value of the cash flows at a given point in time is independent of the path that interest rates followed up to that point. In other words, cash flows are not **path dependent**; cash flows at any node do not depend on the path rates took to get to that node. Because of path dependency of cash flows of mortgage-backed securities, the binomial tree backward induction process cannot be used to value such securities. We instead use the Monte Carlo simulation method to value mortgage-backed securities.

A **Monte Carlo forward-rate simulation** involves randomly generating a large number of interest rate paths, using a model that incorporates a volatility assumption and an assumed probability distribution. A key feature of the Monte Carlo method is that the underlying cash flows can be path dependent.

As with pathwise valuation discussed earlier, the value of the bond is the average of values from the various paths. The simulated paths should be calibrated so benchmark interest rate paths value benchmark securities at their market price (i.e., arbitrage-free valuation). The calibration process entails adding (subtracting) a constant to all rates when the value obtained from the simulated paths is too high (too low) relative to market prices. This calibration process results in a **drift adjusted** model.

A Monte Carlo simulation may impose upper and lower bounds on interest rates as part of the model generating the simulated paths. These bounds are based on the notion of mean reversion; rates tend to rise when they are too low and fall when they are too high.

EXAMPLE: **Valuation of option-free bond using Monte Carlo simulation**

Samuel Favre is interested in valuing the same three-year, 3% annual-pay Treasury bond as discussed before. Favre wants to use Monte Carlo simulation and has generated the following rate paths:

Monte Carlo Simulation (Drift-Adjusted)

Path	Year 1	Year 2	Year 3
1	3%	5.32%	10.59%
2	3%	5.11%	10.33%
3	3%	4.79%	9.89%
4	3%	4.56%	9.10%
5	3%	4.11%	8.22%
6	3%	3.79%	6.54%
7	3%	3.58%	5.11%
8	3%	3.62%	4.11%

Answer:

Valuation using the simulated interest rate paths is shown as follows:

Path	Year 1	Year 2	Year 3	Value
1	3%	5.32%	10.59%	$91.53
2	3%	5.11%	10.33%	$91.91
3	3%	4.79%	9.89%	$92.53
4	3%	4.56%	9.10%	$93.36
5	3%	4.11%	8.22%	$94.47
6	3%	3.79%	6.54%	$96.15
7	3%	3.58%	5.11%	$97.57
8	3%	3.62%	4.11%	$98.42
			Average	$94.49

Where, for example:

$$\text{value}_{\text{path2}} = \frac{3}{(1.03)} + \frac{3}{(1.03)(1.0511)} + \frac{103}{(1.03)(1.0511)(1.1033)} = 91.91$$

Note that the interest rates in the example were calibrated to ensure that the valuation obtained was consistent with market prices (i.e., arbitrage-free) and hence the value obtained is the same as before.

MODULE 26.3: INTEREST RATE MODELS

Video covering this content is available online.

LOS 26.i: Describe term structure models and how they are used.

Term structure models attempt to capture the statistical properties of interest rate movements and provide us with quantitatively precise descriptions of how interest rates will change.

Equilibrium Term Structure Models

Equilibrium term structure models attempt to describe changes in the term structure through the use of fundamental economic variables that drive interest rates. While equilibrium term structure models can rely on multiple factors, the two famous models discussed in the curriculum, the Cox-Ingersoll-Ross (CIR) model and the Vasicek model, are both single-factor models. The single factor in the CIR and Vasicek models is the short-term interest rate.

The Cox-Ingersoll-Ross Model

The **Cox-Ingersoll-Ross (CIR) model** is based on the idea that interest rate movements are driven by individuals choosing between consumption today versus investing and consuming at a later time.

Mathematically, the CIR model consists of two components: the first part of this expression is a drift term, while the second part is the random component:

$$dr_t = a(b - r)\,dt + \sigma\sqrt{r}\,dz$$

where:

dr_t = change in the short-term interest rate
a = speed of mean reversion parameter (a high a means fast mean reversion)
b = long-run value of the short-term interest rate
r = the short-term interest rate
t = time
dt = a small increase in time
σ = volatility
dz = a small random walk movement

The a(b – r)dt term forces the interest rate to mean-revert toward its long-run value (b) at a speed determined by the mean reversion parameter (a).

Under the CIR model, volatility increases with the interest rate, as can be seen in the $\sigma\sqrt{r}dz$ term. In other words, at high interest rates the amount of period-over-period fluctuation in rates is also high.

The Vasicek Model

Like the CIR model, the **Vasicek model** suggests that interest rates are mean reverting to some long-run value.

The Vasicek model is expressed as:

$$dr_t = a(b - r)dt + \sigma dz$$

The difference from the CIR model that you will notice is that no interest rate (r) term appears in the second term σdz, meaning that volatility in this model does not increase as the level of interest rates increases (i.e., volatility is constant).

The main disadvantage of the Vasicek model is that the model does not force interest rates to be nonnegative.

Arbitrage-Free Models

Arbitrage-free models of the term structure of interest rates begin with the assumption that bonds trading in the market are correctly priced, and the model is calibrated to value such bonds consistent with their market price (hence the "arbitrage-free" label). These models do not try to justify the current yield curve; rather, they take this curve as given.

The ability to calibrate arbitrage-free models to match current market prices is one advantage of arbitrage-free models over the equilibrium models.

The Ho-Lee Model

The **Ho-Lee model** takes the following form:

$$dr_t = \theta_t dt + \sigma dz_t$$

where:

θ_t = a time-dependent drift term

Derived using the relative pricing concepts of the Black-Scholes model, this model assumes that changes in the yield curve are consistent with a no-arbitrage condition.

The Ho-Lee model is calibrated by using market prices to find the time-dependent drift term θ_t that generates the current term structure. The Ho-Lee model can then be used to price zero-coupon bonds and to determine the spot curve. The model assumes constant volatility and produces a symmetrical (normal) distribution of future rates.

The Kalotay-Williams-Fabozzi (KWF) Model

The **Kalotay-Williams-Fabozzi (KWF) model** does not assume mean reversion and, like the Ho-Lee model, assumes constant volatility and a constant drift. However, the KWF model assumes that the short-term rate is lognormally distributed. The model takes the following form:

$$d\ln(r_t) = \theta_t dt + \sigma dz_t$$

Note that the right-hand side of the equation is the same as that of the Ho-Lee model. The only difference is that the KWF model assumes lognormal distribution of rates while the Ho-Lee model assumed normal distribution.

Other Models

Gauss+ is a multifactor model that incorporates short-, medium-, and long-term rates, where the long-term rate is designed to be mean reverting and depends on macroeconomic variables. Medium-term rates revert to the long-term rate, while the short-term rate is devoid of a random component—consistent with the role of the central bank controlling the short-term rate.

MODULE QUIZ 26.2, 26.3

Use the following information to answer Questions 1 through 6.

Farah Dane, CFA, works for Geodesic Investing, a small hedge fund that offers investment services for a handful of clients known personally by the owner, Mike DeGrekker. The fund makes few trades, preferring to wait for what it perceives to be arbitrage opportunities before investing. Last year, the fund managed a return of more than 45%, thanks largely to a single transaction on which the company made a profit of $9.4 million.

The transaction, which DeGrekker described as a "valuation farming exercise" involved simultaneously purchasing a government Treasury and selling the corresponding strips for a higher price than the cost of the Treasury.

Dane is currently using a binomial lattice and a pathwise method to value fixed-income bonds in order to identify potential trading opportunities. She has used the binomial lattice shown in Figure 1 to value a three-year, annual pay, 4% coupon risk-free government bond with a par value of $1,000. Her pathwise valuation is also shown.

Figure 1: Binomial Lattice

One-Period Forward Rate in Year

0	1	2
3%	5.7883%	10.7383%
	3.8800%	7.1981%
		4.8250%

Pathwise Valuation

	Year 1	Year 2	Year 3	Value
Path 1	3%	5.7883%	10.7383%	$937.45
Path 2	3%	5.7883%	7.1981%	$965.92
Path 3	3%	5.7883%	4.8250%	$986.07
Path 4	3%	3.8800%	7.1981%	$982.95
Path 5	3%	3.8800%	4.8250%	$1,003.48
			Average	$975.17

Dane is not satisfied with this method of valuation and has put together a report for DeGrekker on the use of the Monte Carlo method, which she feels will lead to more accurate valuations. She quotes the following advantages of using Monte Carlo method:

Advantage 1: The Monte Carlo method will estimate the value of the bond using multiple interest rate paths and hence there is no need to make an assumption about volatility of rates.

Advantage 2: The method could be applied to get more accurate valuations for securities that are interest rate path dependent.

DeGrekker is resistant to the idea as he is concerned about the amount of computing time the model may require. He accepts, however, that the idea of using many paths is attractive. He concedes that, "increasing the number of paths used in the model increases the statistical accuracy of the estimated value and produces a value closer to the true fundamental value of the security."

1. Which of the following statements regarding the valuation of an option-free bond using an arbitrage-free binomial lattice is *most accurate*?
   A. If the binomial lattice is correctly calibrated, it should give the same value for an option-free bond as using the par curve used to calibrate the tree.
   B. The binomial lattice will only produce the same value for an option-free bond as the par curve that was used to calibrate it if the bond is priced at par.
   C. The binomial lattice will only produce the same value for an option-free bond as the par curve that was used to calibrate it if the yield curve is flat.
2. Which of the following statements *most accurately* describes the "valuation farming exercise" undertaken by DeGrekker?
   A. DeGrekker used the process of stripping and the law of one price to make an arbitrage gain.
   B. DeGrekker used the process of reconstitution and the principle of no arbitrage to make a risk-free gain.
   C. DeGrekker's profit is not an arbitrage profit as the securities involved are risk free.
3. Which of the following is *most accurate* regarding the value Dane would obtain using the backward induction method as opposed to the pathwise valuation method for the bond in Figure 1?
   A. Both methods would produce the same value.
   B. The pathwise valuation method will give lower values when interest rates are rising because the backward induction method places a higher weighting on earlier cash flows.

C. The backward induction method will give a different value compared to the pathwise method when the volatility of interest rates is high as the pathwise method uses equal weights.

4. Dane's pathwise valuation is:
   A. correct.
   B. incorrect, as the correct value is lower than \$975.17.
   C. incorrect, as the correct value is higher than \$975.17.

5. Which of the advantages of the Monte Carlo method stated by Dane is *most accurate*?
   A. Advantage 1 only.
   B. Advantage 2 only.
   C. Neither advantage is correct.

6. DeGrekker's comment on increasing the number of paths is *most likely*:
   A. correct.
   B. incorrect in asserting that a larger number of paths will produce an estimate that is statistically more accurate.
   C. incorrect in asserting that a larger number of paths will produce a value closer to the true fundamental value.

7. When calibrating a binomial interest rate tree to match a specific term structure, which of the following statements is *least accurate*?
   A. Interest rates in the tree should produce an arbitrage-free valuation for benchmark securities.
   B. Adjacent spot rates at each node of the tree are related by the multiplier $e^{2\sigma}$.
   C. The middle forward rate in a period is approximately equal to the implied (from the benchmark spot rate curve) one-period forward rate for that period.

8. The modern term structure model that is *most likely* to precisely generate the current term structure is the:
   A. Cox-Ingersoll-Ross model.
   B. Vasicek model.
   C. Ho-Lee model.

KEY CONCEPTS

LOS 26.a

Arbitrage-free valuation leads to a security value such that no market participant can earn an arbitrage profit in a trade involving that security. In other words, the valuation is consistent with the value additivity principle and without dominance of any security relative to others in the market.

LOS 26.b

Arbitrage-free valuation of fixed-rate, option-free bonds entails discounting each of the bond's future cash flows at its corresponding spot rate.

LOS 26.c

The binomial interest rate tree framework is a lognormal model with two equally likely outcomes for one-period forward rates at each node. A volatility assumption drives the spread of the nodes in the tree.

LOS 26.d

A binomial interest rate tree is calibrated such that (1) the values of benchmark bonds using the tree are equal to the bonds' market prices, (2) adjacent forward rates at any nodal period are two standard deviations apart and (3) the midpoint for each nodal period is approximately equal to the implied one-period forward rate for that period.

LOS 26.e

Backward induction is the process of valuing a bond using a binomial interest rate tree. The term backward is used because in order to determine the value of a bond at Node 0, we need to know the values that the bond can take on at nodal period 1, and so on.

LOS 26.f

Valuation of bonds using a zero-coupon yield curve (also known as the spot rate curve) is suitable for option-free bonds. However, for bonds with embedded options where the value of the option varies with outcome of unknown forward rates, a model that allows for variability of forward rates is necessary. One such model is the binomial interest rate tree framework.

LOS 26.g

In the pathwise valuation approach, the value of the bond is simply the average of the values of the bond at each path. For a *n*-period binomial tree, there are $2^{(n-1)}$ possible paths.

LOS 26.h

The Monte Carlo simulation method uses pathwise valuation and a large number of randomly generated simulated paths. Mortgage-backed securities have path-dependent cash flows on account of the embedded prepayment option. The Monte Carlo simulation method should be used for valuing MBS as the binomial tree backward induction process is inappropriate for securities with path-dependent cash flows.

LOS 26.i

Two major classes of term structure models are as follows:

1. Equilibrium term structure models—Attempt to model the term structure using fundamental economic variables that are thought to determine interest rates.

   a. Cox-Ingersoll-Ross model: $dr_t = a(b - r)dt + \sigma\sqrt{r}dz$

   Assumes the economy has a natural long-run interest rate (b) that the short-term rate (r) converges to at a speed of (a). Interest rate volatility varies with *r* and is not constant. Produces non negative rates only.

   b. Vasicek model: $dr_t = a(b - r)dt + \sigma dz$

   Similar to the CIR model, but assumes that the interest rate volatility level is constant and independent of the level of short-term interest rates. Can produce negative rates.

2. Arbitrage-free models—Begin with observed market prices and the assumption that securities are correctly priced.

a. Ho-Lee model: $dr_t = \theta_t dt + \sigma dz_t$

Calibrated by using market prices to find the time-dependent drift term θ_t that generates the current term structure. Assumes that short-term rates are normally distributed with a constant volatility. Produces a normal distribution of rates and rates can be negative.

b. Kalotay-Williams-Fabozzi (KWF) model: $d\ln(r_t) = \theta_t dt + \sigma dz_t$

Similar to the Ho-Lee model, but assumes that short rate has a lognormal distribution.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 26.1

1. **B** An arbitrage gain is a risk-free profit and hence requires no net investment. The returns therefore are not simply the risk-free rate. As there is no initial investment, the gains cannot be measures as percentage of initial cost. (LOS 26.a)

2. **B** An up-front arbitrage profit of $38.70 can be earned by selling 10 FFPQ bonds short and purchasing 1 DALO and 1 NKDS bonds as shown here.

Position	Initial Cash Flow	Year 1 Cash Flow	Year 2 Cash Flow
Short 10 FFPQ	$ 11,701.20	$(1,000.00)	$(11,000.00)
Long 1 DALO	$ (985.22)	$ 1,000.00	$ -
Long 1 NKDS	$(10,667.28)	$ -	$ 11,000.00
Net	$38.70	0	0

(LOS 26.a)

3. **A** FFPQ is overpriced. Based on the 1.5% benchmark yield, the other two bonds are correctly priced.

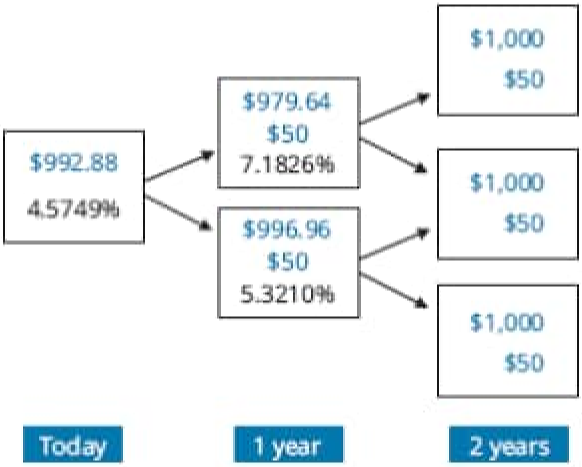
Arbitrage-free price = (PV Year 1 Cash Flow) + (PV Year 2 Cash Flow)

	PV Year 1 Cash Flow	PV Year 2 Cash Flow	Arbitrage-Free Price
FFPQ	98.52	1,067.73	1,166.25
DALO	985.22	0	985.22
NKDS	0	10,677.28	10,677.28

(LOS 26.a)

4. **A** A lognormal random walk will ensure non-negativity but will lead to higher volatility at higher interest rates. (LOS 26.c)

5. **C** The value of the 5%, two-year annual pay $1000 par bond is $992.88.



(LOS 26.e)

Module Quiz 26.2, 26.3

1. **A** A correctly calibrated tree will value the bond at the same price as the par and spot curves used to derive it. (Module 26.2, LOS 26.d)

2. **A** DeGrekker purchased the bonds and stripped them into constituent parts before selling them. The strategy involved no initial net investment and yet results in an arbitrage profit. (Module 26.1, LOS 26.a)

3. **A** The two methods are identical and will always give the same result. (Module 26.2, LOS 26.g)

4. **B** This is a tricky question. There are only four possible paths that Dane should have used. The possible paths are UU (Path 1), UD (Path 2), DU (Path 4), and DD (Path 5). Path 3 isn't a valid path.

Path	Year 1	Year 2	Year 3
Path 1	3%	5.7883%	10.7383%
Path 2	3%	5.7883%	7.1981%
Path 4	3%	3.8800%	7.1981%
Path 5	3%	3.8800%	4.8250%

So the value should be the average of the values for Paths 1, 2, 4, and 5. The correct average value is $972.45. (Module 26.2, LOS 26.g)

5. **B** Monte Carlo simulation also requires an assumed level of volatility as an input. (Module 26.2, LOS 26.h)

6. **C** The larger the number of paths, the more accurate the value in a statistical sense. However, whether the value is closer to the true fundamental value depends on the accuracy of the model inputs. (Module 26.2, LOS 26.h)

7. **B** The stated multiplier is correct but it is important to note that the rates given at each node of the tree are forward rates not spot rates. (Module 26.2, LOS 26.d)

8. **C** The Ho-Lee model is calibrated by using market prices to find the time-dependent drift term, θ_t that generates the current term structure. (One of the drawbacks of the Vasicek and Cox-Ingersoll-Ross models is that the model prices generated by these models generally do not coincide with observed market prices.) (Module 26.3, LOS 26.i)

READING 27

VALUATION AND ANALYSIS OF BONDS WITH EMBEDDED OPTIONS

EXAM FOCUS

This topic review extends the arbitrage-free valuation framework to valuation of bonds with embedded options. Understand the risk/return dynamics of embedded options, including their impact on a bond's duration and convexity. You should also know the adjustment required for the valuation of credit-risky bonds, including the process to estimate an OAS. Finally, understand the terminology and risk/return characteristics of convertibles.

MODULE 27.1: TYPES OF EMBEDDED OPTIONS

Video covering this content is available online.

LOS 27.a: Describe fixed-income securities with embedded options.

Embedded options in a bond allow an issuer to (1) manage interest rate risk and/or (2) issue the bonds at an attractive coupon rate. The embedded options can be a simple call or put option, or more complex options such as provisions for a sinking fund or an estate put.

Simple Options

Callable bonds give the *issuer* the option to call back the bond; the *investor* is *short* the call option. Most callable bonds have a **call protection period** during which the bond cannot be called. The call option can be a **European-style** option (whereby the option can only be exercised on a single day immediately after the protection period), an **American-style** option (whereby the option can be exercised at any time *after* the protection period), or even a **Bermudan-style** option (whereby the option can be exercised at fixed dates after the protection period).

Putable bonds allow the *investor* to put (sell) the bond back to the issuer prior to maturity. The *investor* is *long* the underlying put option. A related bond is an **extendible bond**, which allows the investor to extend the maturity of the bond. An extendible bond can be evaluated as a putable bond with longer maturity (i.e., the maturity if the bond is extended). A two-year, 3% bond extendible for an additional

year at the same coupon rate would be valued the same as an otherwise identical three-year putable (European style) bond with a protection period of two years.

Complex Options

More complex options include:

- An **estate put** which includes a provision that allows the heirs of an investor to put the bond back to the issuer upon the death of the investor. The value of this contingent put option is inversely related to the investor's life expectancy; the shorter the life expectancy, the higher the value.
- **Sinking fund bonds** (sinkers) which require the issuer to set aside funds periodically to retire the bond (a sinking fund). This provision reduces the credit risk of the bond. Sinkers typically have several related *issuer* options (e.g., call provisions, acceleration provisions, and delivery options).

LOS 27.b: Explain the relationships between the values of a callable or putable bond, the underlying option-free (straight) bond, and the embedded option.

In essence, the holder of a callable bond owns an option-free (straight) bond and is also short a call option written on the bond. The value of the embedded call option (V_{call}) is, therefore, simply the difference between the value of a straight ($V_{straight}$) bond and the value of the comparable callable bond ($V_{callable}$):

$$V_{call} = V_{straight} - V_{callable}$$

Conversely, investors are willing to pay a premium for a putable bond since its holder effectively owns an option-free bond plus a put option. The value of a putable bond can be expressed as:

$$V_{putable} = V_{straight} + V_{put}$$

Rearranging, the value of the embedded put option can be stated as:

$$V_{put} = V_{putable} - V_{straight}$$

Video covering this content is available online.

MODULE 27.2: VALUING BONDS WITH EMBEDDED OPTIONS, PART 1

LOS 27.c: Describe how the arbitrage-free framework can be used to value a bond with embedded options.

LOS 27.f: Calculate the value of a callable or putable bond from an interest rate tree.

The basic process for valuing a callable (or putable) bond is similar to the process for valuing a straight bond. However, instead of using spot rates, one-period forward rates are used in a binomial tree framework. This change in methodology computes the value of the bond at different points in time; these checks are necessary to determine whether the embedded option is in-the-money (and exercised).

When valuing a callable bond, the value at any node where the bond is callable must be either the price at which the issuer will call the bond (i.e., the call price) or the computed value if the bond is not called, *whichever is lower*. This is known as the call rule. Similarly, for a putable bond, the value used at any node corresponding to a put date must be either the price at which the investor will put the bond (i.e., the put price) or the computed value if the bond is not put, *whichever is higher*. This is known as the put rule.

PROFESSOR'S NOTE
Call date and put date in this context vary depending on whether the option is European-, American-, or Bermudan-style.

EXAMPLE: Valuation of call and put options

Consider a two-year, 7% annual-pay, $100 par bond callable in one year at $100. Also consider a two-year, 7% annual-pay, $100 par bond putable in one year at $100.

The interest rate tree at 15% assumed volatility is as follows:

Value the embedded call and put options.

Answer:

Value of the straight (option-free) bond:

Consider the value of the bond at the *upper* node for Period 1, $V_{1,U}$:

$$V_{1,U} = \frac{1}{2} \times \left[\frac{\$100 + \$7}{1.071826} + \frac{\$100 + \$7}{1.071826}\right] = \$99.830$$

Similarly, the value of the bond at the *lower* node for Period 1, $V_{1,L}$, is:

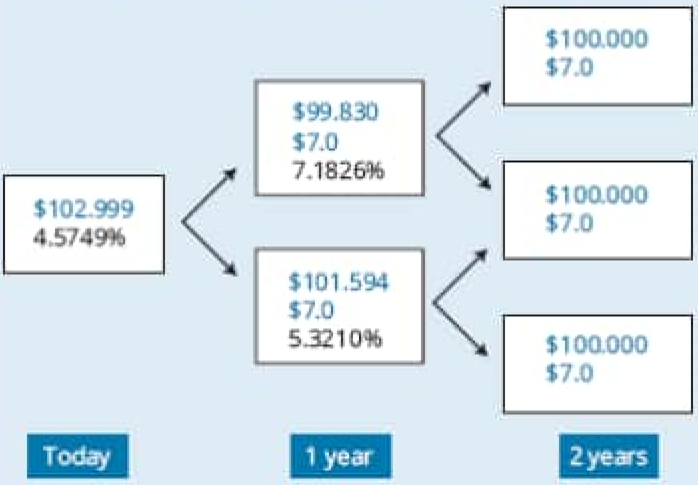
$$V_{1,L} = \frac{1}{2} \times \left[\frac{\$100 + \$7}{1.053210} + \frac{\$100 + \$7}{1.053210}\right] = \$101.594$$

Now calculate V_0, the current value of the bond at Node 0.

$$V_0 = \frac{1}{2} \times \left[\frac{\$99.830 + \$7}{1.045749} + \frac{\$101.594 + \$7}{1.045749}\right] = \$102.999$$

The completed binomial tree is shown as follows:

Valuing a Two-Year, 7.0% Coupon, Option-Free Bond



Value of the callable bond:

The call rule (call the bond if the price exceeds $100) is reflected in the boxes in the completed binomial tree, where the second line of the boxes at the one-year node is the lower of the call price or the computed value. For example, the value of the bond in one year at the lower node is $101.594. However, in this case, the bond will be called, and the investor will only receive $100. Therefore, for valuation purposes, the value of the bond in one year at this node is $100.

$V_{1,L} = \$100$

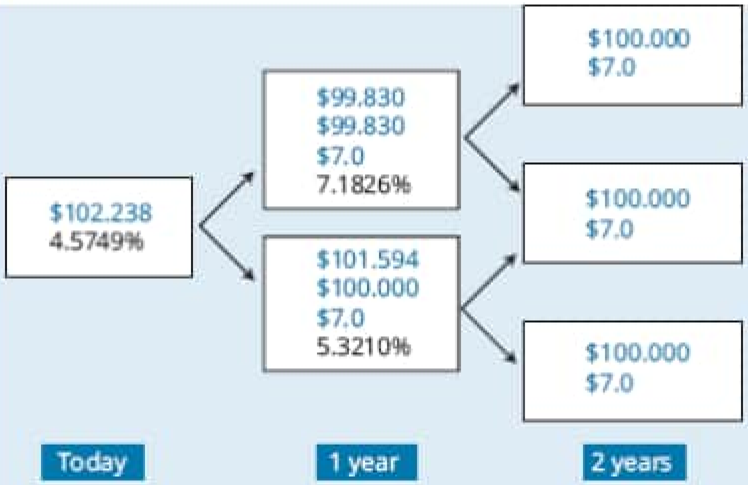
$V_{1,U} = (107 / 1.071826) = \$99.830$

The calculation for the current value of the bond at Node 0 (today), assuming the simplified call rules of this example, is:

$$V_0 = \frac{1}{2} \times \left[\frac{\$99.830 + \$7}{1.045749} + \frac{\$100.00 + \$7}{1.045749}\right] = \$102.238$$

The completed binomial tree is shown as follows:

Valuing a Two-Year, 7.0% Coupon, Callable Bond, Callable in One Year at 100



Value of the putable bond:

Similarly, for a putable bond, the put rule is to put the bond if the value falls below $100. The put option would therefore be exercised at the upper-node in Year 1 and hence the $99.830 computed value is replaced by the exercise price of $100.

$V_{1,U} = 100$

$V_{1,L} = (107 / 1.053210) = \$101.594$

$$V_0 = \frac{1}{2} \times \left[\frac{100 + 7}{1.045749} + \frac{101.594 + 7}{1.045749}\right] = \$103.081$$

Value of the embedded options:

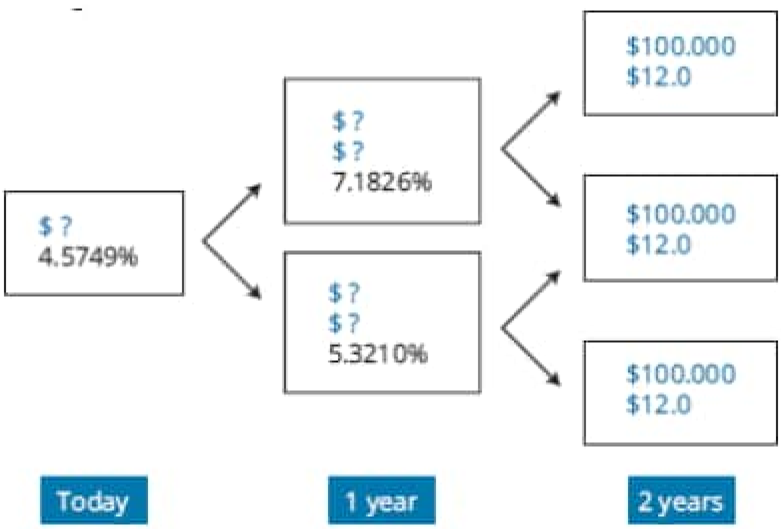
$V_{call} = V_{straight} - V_{callable} = \$102.999 - \$102.238 = \$0.76$

$V_{put} = V_{putable} - V_{straight} = \$103.081 - \$102.999 = 0.082$

MODULE QUIZ 27.1, 27.2

1. Which of the following statements concerning the calculation of value at a node in a binomial interest rate tree is *most accurate*? The value at each node is the:
   A. present value of the two possible values from the next period.
   B. average of the present values of the two possible values from the next period.
   C. sum of the present values of the two possible values from the next period.

Use the following binomial interest rate tree to answer Questions 2 through 4.



2. The value today of an option-free, 12% annual coupon bond with two years remaining until maturity is *closest* to:
   A. 110.525.
   B. 111.485.
   C. 112.282.
3. The value of the bond and the value of the embedded call option, assuming the bond in Question 2 is callable at $105 at the end of Year 1, are *closest* to:

	Callable bond value	Embedded call option value
A.	110.573	1.709
B.	110.573	0.642
C.	111.640	0.642

4. The value of the bond and the value of the embedded put option, assuming the bond in Question 2 is putable at $105 at the end of Year 1, are *closest* to:

	Putable bond value	Embedded put option value
A.	112.523	0.241
B.	112.523	1.646
C.	113.928	1.646

Video covering this content is available online.

MODULE 27.3: VALUING BONDS WITH EMBEDDED OPTIONS, PART 2

LOS 27.d: Explain how interest rate volatility affects the value of a callable or putable bond.

Option values are positively related to the volatility of their underlying. Accordingly, when interest rate volatility increases, the values of both call and put options increase. The value of a straight bond is affected by changes in the level of interest rates but is *unaffected* by changes in the *volatility* of interest rates.

When interest rate volatility increases, the value of a callable bond (where the investor is short the call option) decreases and the value of a putable bond (where the investor is long the put option) increases.

LOS 27.e: Explain how changes in the level and shape of the yield curve affect the value of a callable or putable bond.

Level of Interest Rates

As interest rates decline, the short call in a callable bond limits the bond's upside, so the value of a callable bond rises less rapidly than the value of an otherwise-equivalent straight bond.

As interest rates increase, the long put in a putable bond hedges against the loss in value; the value of a putable bond falls less rapidly than the value of an otherwise-equivalent straight bond.

Call option value is inversely related to the level of interest rates, while put option value varies directly with the level of interest rates.

Shape of the Yield Curve

The value of an embedded call option increases as interest rates decline. When the yield curve is upward sloping (i.e., normal), the more distant one-period forward rates are higher than the one-period forward rates in the near future. Because a higher interest rate scenario limits the probability of the call option being in the money, the value of a call option will be lower for an upward sloping yield curve. As an upward-sloping yield curve becomes flatter, the call option value increases.

The value of a put option increases with interest rates. When the yield curve is upward sloping, the probability of the put option going in the money is higher. Put option value therefore declines as an upward-sloping yield curve flattens.

MODULE 27.4: OPTION-ADJUSTED SPREAD

Video covering this content is available online.

LOS 27.g: Explain the calculation and use of option-adjusted spreads.

So far our backward induction process has relied on the risk-free binomial interest rate tree; our valuation assumed that the underlying bond was risk-free. If risk-free rates are used to discount cash flows of a credit risky corporate bond, the calculated value will be too high. To correct for this, a constant spread must be added to all one-period rates in the tree such that the calculated value equals the market price of the risky bond. This constant spread is called the **option adjusted spread (OAS)**.

PROFESSOR'S NOTE

The OAS is added to the tree after the adjustment for the embedded option (i.e., the node values are adjusted according to the call/put rule). Hence the OAS is calculated after the option risk has been removed.

EXAMPLE: Computation of OAS

A $100-par, three-year, 6% annual-pay ABC, Inc., callable bond trades at $99.95. The underlying call option is a Bermudan-style option that can be exercised in one or two years at par.

The benchmark interest rate tree assuming volatility of 20% is provided here:

One-Period Forward Rates		
Year 0	Year 1	Year 2
3.000%	5.7883%	10.7383%
	3.8800%	7.1981%
		4.8250%

Compute the OAS on the bond.

Answer:

The value of the bond using the benchmark interest rate tree is $101.77, as shown here:

To force the computed value to be equal to the current market price of $99.95, a constant spread (OAS) of 100 bps is added to each interest rate in the tree, as shown here:

PROFESSOR'S NOTE

The OAS computed using the methodology just listed is the spread implied by the current market price and hence assumes that the bond is priced correctly. Note that the actual estimation of OAS is largely an iterative process and is beyond the scope of the exam.

OAS is used by analysts in relative valuation; bonds with similar credit risk should have the same OAS. If the OAS for a bond is higher than the OAS of its peers, it is considered to be undervalued and hence an attractive investment (i.e., it offers a higher compensation for a given level of risk). Conversely, bonds with low OAS (relative to peers) are considered to be overvalued.

LOS 27.h: Explain how interest rate volatility affects option-adjusted spreads.

Consider for a moment the difference between the calculated value of a bond that we obtain from a tree and the bond's actual market price. The greater this difference is, the greater the OAS we would need to add to the rates in the tree to force the calculated bond value down to the market price.

Suppose that a 7%, 10-year callable bond of XYZ Corp. is trading for $958.

Analyst A assumes a 15% value for future volatility in generating her *benchmark* interest rate tree and calculates the value of the bond as $1,050. She then computes the OAS (the increase in discount rate required to lower the calculated bond value to the market price of $958) for this bond to be 80 bps.

Analyst B assumes a 20% value for future volatility, and because of higher assumed volatility, the computed value of the bond turns out to be $992. (Note that this is lower than the $1,050 calculated by A—the higher the volatility, the lower a callable bond's value.) The OAS calculated by B is accordingly lower at 54 bps.

Observe that for the same bond, the OAS calculated varied depending on the volatility assumption used.

When we use a higher estimate of volatility to value a callable bond, the calculated value of the call option increases, the calculated value of the straight bond is unaffected, and the *computed value* (not the market price) of the callable bond

decreases (since the bondholder is short the option). Hence when the estimated (or assumed) volatility (of benchmark rates) used in a binomial tree is higher, the computed value of a callable bond will be lower—and therefore closer to its true market price. The constant spread that needs to be added to the benchmark rates to correctly price the bond (the OAS) is therefore lower.

Figure 27.1: Relationship Between Volatility and OAS

Assumed Level of Volatility	Value				OAS_{CALL}	OAS_{PUT}
	Calls	Puts	Callable	Putable		
High	High	High	Low	High	Low	High
Low	Low	Low	High	Low	High	Low

To summarize, as the *assumed* level of volatility used in an interest rate tree increases, the computed OAS (for a given market price) for a callable bond decreases. Similarly, the computed OAS of a putable bond increases as the assumed level of volatility in the binomial tree increases.

PROFESSOR'S NOTE

Notice that the columns for the value of callable and putable bonds in the preceding figure match the corresponding OAS columns.

MODULE QUIZ 27.3, 27.4

1. The option adjusted spread (OAS) on a callable corporate bond is 73 basis points using on-the-run Treasuries as the benchmark rates in the construction of the binomial tree. The *best* interpretation of this OAS is the:
   A. cost of the embedded option is 73 basis points.
   B. cost of the option is 73 basis points over Treasury.
   C. spread that reflects the credit risk is 73 basis points over Treasury.
2. An increase in interest rate volatility increases the value of:
   A. bonds with embedded call options.
   B. bonds with embedded put options.
   C. low-coupon bonds with embedded options, but decreases the value of high-coupon bonds with embedded options.

MODULE 27.5: DURATION

Video covering this content is available online.

LOS 27.i: Calculate and interpret effective duration of a callable or putable bond.

Recall from Level I that:

- Modified duration measures a bond's price sensitivity to interest rate changes, *assuming that the bond's cash flows do not change as interest rates change.*
- The standard measure of convexity can be used to improve price changes estimated from modified duration.

Modified duration and convexity are not useful for bonds with embedded options, however, because the cash flows from these bonds will change if the option is exercised. To overcome this problem, ***effective* duration** and ***effective* convexity**

should be used, because these measures take into account how changes in interest rates may alter cash flows.

The following expressions can be used to compute effective duration and effective convexity for *any* bond:

$$\text{effective duration} = \text{ED} = \frac{\text{BV}_{-\Delta y} - \text{BV}_{+\Delta y}}{2 \times \text{BV}_0 \times \Delta y}$$

$$\text{effective convexity} = \text{EC} = \frac{\text{BV}_{-\Delta y} + \text{BV}_{+\Delta y} - (2 \times \text{BV}_0)}{\text{BV}_0 \times \Delta y^2}$$

where:

Δy = change in required yield, in decimal form

$\text{BV}_{-\Delta y}$ = estimated price if yield decreases by Δy

$\text{BV}_{+\Delta y}$ = estimated price if yield increases by Δy

BV_0 = initial observed bond price

Calculating effective duration and effective convexity for bonds with embedded options is a complicated undertaking because we must calculate values of $\text{BV}_{+\Delta y}$ and $\text{BV}_{-\Delta y}$. Here's how it is done:

Step 1: Given assumptions about benchmark interest rates, interest rate volatility, and any calls and/or puts, calculate the OAS for the issue using the current market price and the binomial model.

Step 2: Impose a small parallel shift in the benchmark yield curve by an amount equal to $+\Delta y$.

Step 3: Build a new binomial interest rate tree using the new yield curve.

Step 4: Add the OAS from step 1 to each of the one-year rates in the interest rate tree to get a "modified" tree.

Step 5: Compute $\text{BV}_{+\Delta y}$ using this modified interest rate tree.

Step 6: Repeat steps 2 through 5 using a parallel rate shift of $-\Delta y$ to obtain a value of $\text{BV}_{-\Delta y}$.

LOS 27.j: Compare effective durations of callable, putable, and straight bonds.

Both call and put options have the potential to reduce the life of a bond, so the duration of callable and putable bonds will be less than or equal to the duration of their straight counterparts.

- Effective duration (callable) ≤ effective duration (straight).
- Effective duration (putable) ≤ effective duration (straight).
- Effective duration (zero-coupon) ≈ maturity of the bond.
- Effective duration of fixed-rate coupon bond < maturity of the bond.
- Effective duration of floater ≈ time (in years) to next reset.

While effective duration of straight bonds is relatively unaffected by changes in interest rates, an increase in rates would increase the effective duration of a callable

bond, and decrease the effective duration of a putable bond.

MODULE QUIZ 27.5

1. Ron Hyatt has been asked to do a presentation on how effective duration (ED) and effective convexity (EC) are calculated with a binomial model. His presentation includes the following formulas:

$$\text{effective duration} = \text{ED} = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$$

$$\text{effective convexity} = \text{EC} = \frac{BV_{-\Delta y} + BV_{+\Delta y} - (2 \times BV_0)}{2 \times BV_0 \times \Delta y^2}$$

where:

Δy = change in required yield, in decimal form

$BV_{-\Delta y}$ = estimated price if yield decreases by Δy

$BV_{+\Delta y}$ = estimated price if yield increases by Δy

BV_0 = initial observed bond price

Are Hyatt's formulas for effective duration and effective convexity correctly presented?

A. The formulas are both correct.
B. One formula is correct, the other incorrect.
C. Both formulas are incorrect.

MODULE 27.6: KEY RATE DURATION

Video covering this content is available online.

LOS 27.k: Describe the use of one-sided durations and key rate durations to evaluate the interest rate sensitivity of bonds with embedded options.

One-Sided Durations

The computation of effective duration discussed earlier relied on computing the value of the bond for equal parallel shifts of the yield curve up and down (by the same amount). This metric captures interest rate risk reasonably well for small changes in the yield curve and for option-free bonds.

For a callable bond, when the call option is at or near the money, the change in price for a decrease in yield will be less than the change in price for an equal amount of increase in yield. The value of a callable bond is capped by its call price: the bond's value will not increase beyond the call price regardless of how low interest rates fall. Similarly, the value of a putable bond is more sensitive to downward movements in yield curve versus upward movements.

For bonds with embedded options, **one-sided durations**—durations that apply only when interest rates rise (or, alternatively, only when rates fall)—are better at capturing interest rate sensitivity than simple effective duration. When the underlying option is at-the-money (or near-the-money), callable bonds will have lower one-sided down-duration than one-sided up-duration: the price change of a callable when rates fall is smaller than the price change for an equal increase in

rates. Conversely, a near-the-money putable bond will have larger one-sided down-duration than one-sided up-duration.

Key Rate Duration

Key rate durations or **partial durations** capture the interest rate sensitivity of a bond to changes in yields (par rates) of specific benchmark maturities. Key rate duration is used to identify the interest rate risk from changes in the shape of the yield curve (shaping risk).

The process of computing key rate duration is similar to the process of computing effective duration described earlier, except that instead of shifting the entire benchmark yield curve, only one specific par rate (key rate) is shifted before the price impact is measured.

Figure 27.2 shows the key rate durations for several 15-year *option-free* bonds, each with a different coupon rate and YTM = 3%.

Figure 27.2: Key Rate Durations of Various 15-Year Option-Free Bonds With Different Coupon Rates

Coupon	Price	Key Rate Durations					
		Total	2-year	3-year	5-year	10-year	15-year
1%	$76.12	13.41	−0.05	−0.07	−0.22	−0.45	14.20
2%	$88.06	12.58	−0.03	−0.05	−0.15	−0.27	13.08
3%	$100.00	11.94	0	0	0	0	11.94
5%	$123.88	11.03	0.02	0.1	0.15	0.32	10.44
8%	$159.69	10.18	0.09	0.15	0.32	0.88	8.74

PROFESSOR'S NOTE

In these figures the market interest rate is not changing; rather we are considering a number of different bonds that have different coupons.

Figure 27.3 shows the key rate durations for several 15-year European-style *callable* bonds, each with a different coupon rate (callable in 10 years at par).

Figure 27.3: Key Rate Durations of Various 15-Year Callable Bonds With Different Coupon Rates

Coupon	Price	Key Rate Durations					
		Total	2-year	3-year	5-year	10-year	15-year
1%	$75.01	13.22	−0.03	−0.01	−0.45	−2.22	15.93
2%	$86.55	12.33	−0.01	−0.03	−0.15	5.67	6.85
3%	$95.66	11.45	0.00	0.00	0.00	6.40	5.05
5%	$112.87	9.22	0.02	0.10	0.15	6.67	2.28
8%	$139.08	8.89	0.09	0.15	0.32	7.20	1.13

Figure 27.4 shows the key rate durations for several 15-year European-style *putable* bonds, each with a different coupon rate (putable in 10 years at par).

Figure 27.4: Key Rate Durations of Various 15-Year Putable Bonds With Different Coupon Rates

Coupon	Price	Key Rate Durations					
		Total	2-year	3-year	5-year	10-year	15-year
1%	$77.24	9.22	–0.03	–0.01	–0.45	8.66	1.05
2%	$89.82	9.90	–0.01	–0.03	–0.15	7.23	2.86
3%	$95.66	10.50	0.00	0.00	0.00	5.12	5.38
5%	$123.88	10.70	0.02	0.10	0.15	2.89	7.54
8%	$159.69	10.08	0.09	0.15	0.32	0.45	9.07

The following generalizations can be made about key rates:

1. If an option-free bond is trading at par, the bond's maturity-matched rate is the only rate that affects the bond's value. Its maturity key rate duration is the same as its effective duration, and all other rate durations are zero. In Figure 27.2, the 3% bond's (i.e., the par bond) 15-year key rate duration is same as the bond's effective duration, and all other rate durations are zero.
2. For an option-free bond *not* trading at par, the maturity-matched rate is still the most important rate. In Figure 27.2, the 15-year key rate duration is highest.
3. A bond with a low (or zero) coupon rate may have negative key rate durations for horizons other than the bond's maturity. This is evidenced by some negative key rate durations for 1% and 2% coupon bonds in all three figures.
4. A callable bond with a low coupon rate is unlikely to be called; hence, the bond's maturity-matched rate is the most critical rate (i.e., the highest key rate duration corresponds to the bond's maturity). For the 1% bond in Figure 27.3, the 15-year key rate duration exceeds all other key rate durations.
5. All else equal, higher coupon bonds are more likely to be called, and therefore the time-to-exercise rate will tend to dominate the time-to-maturity rate. For the 8% coupon bond in Figure 27.3, the 10-year key rate duration is highest.
6. Putable bonds with high coupon rates are unlikely to be put, and thus are most sensitive to their maturity-matched rates. For the 8% bond in Figure 27.4, the 15-year key rate duration is the highest.
7. All else equal, lower coupon bonds are more likely to be put, and therefore the time-to-exercise rate will tend to dominate the time-to-maturity rate. For the 1% coupon bond in Figure 27.4, the 10-year key rate duration is highest.

LOS 27.l: Compare effective convexities of callable, putable, and straight bonds.

Straight bonds have positive effective convexity: the increase in the value of an option-free bond is higher when rates fall than the decrease in value when rates increase by an equal amount. When rates are high, callable bonds are unlikely to be called and will exhibit positive convexity. When the underlying call option is near the money, its effective convexity turns negative; the upside potential of the bond's price is limited due to the call (while the downside is not protected). Putable bonds exhibit positive convexity throughout.

MODULE 27.7: CAPPED AND FLOORED FLOATERS

LOS 27.m: Calculate the value of a capped or floored floating-rate bond.

Video covering this content is available online.

A floating-rate bond ("floater") pays a coupon that adjusts every period based on an underlying reference rate. The coupon is typically paid in arrears, meaning the coupon rate is *determined* at the beginning of a period but is *paid* at the end of that period.

A **capped floater** effectively contains an issuer option that prevents the coupon rate from rising above a specified maximum rate known as the **cap**.

value of a capped floater
= value of a "straight" floater − value of the embedded cap

A related floating rate bond is the **floored floater**, which has a coupon rate that will not fall below a specified minimum rate known as the **floor**. In this instance, the embedded option belongs to the investor and offers protection from falling interest rates.

value of a floored floater
= value of a "straight" floater + value of the embedded floor

We can use the standard backward induction methodology in a binomial interest rate tree to value a capped or floored floater. As with the valuation of a bond with embedded options, we must adjust the value of the floater at each node to reflect the exercise of an in-the-money option (in this case, a cap or a floor).

EXAMPLE: **Value of a capped and floored floating-rate bond**

Susane Albright works as a fixed income analyst with Zedone Banks, NA. She has been asked to value a $100 par, two-year, floating-rate note that pays MRR (set in arrears). The underlying bond has the same credit quality as reflected in the swap curve. Albright has constructed the following two-year binomial MRR tree:

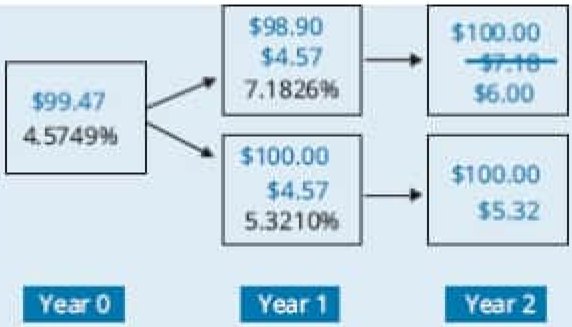
One-Period Forward Rate	
Year 0	Year 1
4.5749%	7.1826%
	5.3210%

How would we compute the following?

1. The value of the floater, assuming that it is an option-free bond.
2. The value of the floater, assuming that it is capped at a rate of 6%. Also compute the value of the embedded cap.
3. The value of the floater, assuming that it is floored at a rate of 5%. Also compute the value of the embedded floor.

Answer:

1. An option-free bond with a coupon rate equal to the required rate of return will be worth par value. Hence, the straight value of the floater is $100.
2. The value of the capped floater is $99.47, as shown here:



The upper node in Year 2 shows the exercise of the cap (the coupon is capped at $6.00 instead of rising to $7.18).

Note that when the option is not in the money, the floater is valued at par.

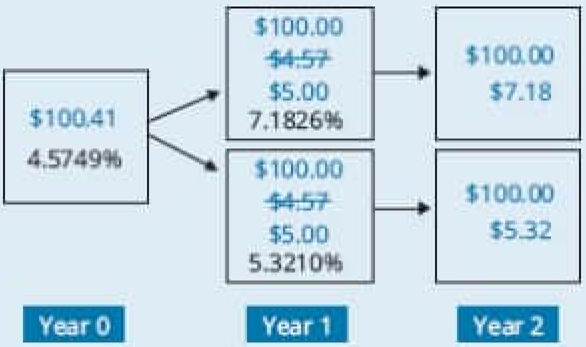
$$V_{1,U} = (\$100 + \$6) / (1 + 0.071826) = \$98.90$$
$$V_{1,L} = (100 + 5.3210) / (1 + 0.05321) = \$100$$

The Year 0 value is the average of the Year 1 values (including their adjusted coupons) discounted for one period. In this case, the Year 1 coupons require no adjustment, as the coupon rate is below the cap rate.

$$V_0 = \frac{[(98.90 + 4.57) + (100 + 4.57)] / 2}{(1.045749)} = \$99.47$$

Thus the value of the embedded cap = $100 – $99.47 = 0.53.

3. The value of the floored floater is $100.41, as shown here:



The nodes for Year 2 show the coupons for that period (none of the rates are below the floor, and hence the floor is not exercised). Strikethroughs for both nodes in Year 1 indicate that the floor was in the money; we replace the MRR = market reference rate. Having another rate after it is redundant coupon with a coupon based on the floor strike rate of 5%.

The Year 0 value is the average of the Year 1 values (including their adjusted coupons) discounted for one period:

$$V_0 = \frac{[(100 + 5) + (100 + 5)] / 2}{(1.045749)} = \$100.41$$

Thus the value of the embedded floor = $100.41 – $100 = $0.41.

MODULE QUIZ 27.6, 27.7

Use the following information to answer Questions 1 through 8.

Vincent Osagae, CFA, is the fixed income portfolio manager for Alpha Specialists, an institutional money manager. Vanessa Alwan, an intern, has suggested a list of bonds

for Osagae's consideration as shown in Figure 1. The benchmark yield curve is currently upward sloping.

Figure 1: Selected Bonds for Consideration

- Bond X, a 3%, 15-year option-free bond.
- Bond Y, a 3%, 15-year callable bond.
- Bond Z, a 3%, 15-year putable bond.

Osagae then turns his attention to a newly issued 4%, 15-year bond issued by Suni Corp. The bond has a Bermudan-style call option exercisable annually at any time after Year 4. Osagae computes an OAS of 145 bps for this bond using a binomial interest rate tree. The tree was generated with U.S. Treasury rates and an assumed volatility of 15%, which was consistent with historical data. Option markets are currently pricing interest rate options at an implied volatility of 19%.

Every week, Alwan meets with Osagae to discuss what she had learned over the week. At one of their weekly meetings, Alwan makes the following statements about key rate duration:

Statement 1: The highest key rate duration of a low-coupon callable bond corresponds to the bond's time-to-exercise.

Statement 2: The highest key rate duration of a high-coupon putable bond corresponds to the bond's time-to-maturity.

Beth Grange, a senior analyst reporting to Osagae has generated the following two-year MRR tree using a 15% volatility assumption:

Figure 2: MRR Tree

0	1
2.0000%	3.4637%
	2.5660%

Grange has also compiled OAS for two very similar callable corporate bonds as shown in Figure 3.

Figure 3: OAS at 10% Volatility

Bond	OAS
A	46 bps
B	34 bps

1. In Figure 1, which bond's embedded option is *most likely* to increase in value if the yield curve flattens?
   A. Bond Y.
   B. Bond Z.
   C. Neither Bond Y nor Bond Z.
2. If Osagae had used implied volatility from interest rate options to compute the OAS for Suni Corp bond, the estimated OAS would *most likely* have been:
   A. lower than 145 bps.
   B. higher than 145 bps.
   C. equal to 145 bps.
3. Which bond in Figure 1 is *most likely* to have the highest effective duration?
   A. Bond X.
   B. Bond Y.
   C. Bond Z.

4. Regarding Alwan's statements about key rate durations:
   A. only one statement is correct.
   B. both statements are correct.
   C. neither statement is correct.
5. Which bond in Figure 1 is *least likely* to experience the highest increase in value, given a parallel downward shift of 150 bps in the yield curve?
   A. Bond X.
   B. Bond Y.
   C. Bond Z.
6. Which bond in Figure 1 is *most likely* to experience an increase in effective duration due to an increase in interest rates?
   A. Bond X.
   B. Bond Y.
   C. Bond Z.
7. In Figure 3, relative to bond A, bond B is *most likely*:
   A. overpriced.
   B. underpriced.
   C. fairly priced.
8. Using the data in Figure 2, the value of a $100 par, two-year, 3% capped floater is *closest* to:
   A. $0.31.
   B. $98.67.
   C. $99.78.

MODULE 27.8: CONVERTIBLE BONDS

Video covering this content is available online.

LOS 27.n: Describe defining features of a convertible bond.

The owner of a **convertible bond** has the right to convert the bond into a fixed number of common shares of the issuer during a specified timeframe (**conversion period**) and at a fixed amount of money (**conversion price**). Convertibles allow investors to enjoy the upside on the issuer's stock, although this comes at a cost of lower yield. The issuer of a convertible bond benefits from a lower borrowing cost, but existing shareholders may face dilution if the conversion option is exercised.

The **conversion ratio** is the number of common shares for which a convertible bond can be exchanged. For example, a convertible bond issued at par with an initial conversion ratio of 10 allows its holder to convert one $1,000 par bond into 10 shares of common stock. Equivalently, the **conversion price** of the bond is $1,000 / 10 shares = $100. For bonds not issued at par, the conversion price is the issue price divided by the conversion ratio. Offer documents will indicate how the initial conversion ratio would be modified to account for corporate actions such as stock splits or stock dividends.

Offer documents may also provide a contingent put option in the event of any change-of-control events such as mergers. Such a contingent put option can be exercised for a specific period of time after the change of control. Alternatively, a lower conversion price may be specified in the event of a change of control. The conversion ratio may also be adjusted upward if the company pays a dividend in excess of a specified **threshold dividend**. This adjustment protects the convertible

bondholders in the event that the company pays out an unusually large dividend (which will cause the ex-dividend price of the stock to decline). Other put options exercisable during specific periods may also be embedded with a convertible. These put options can be hard puts (i.e., redeemable for cash) or soft puts (i.e., the issuer decides whether to redeem the bond for cash, stock, subordinated debentures, or a combination of the three).

LOS 27.o: Calculate and interpret the components of a convertible bond's value.

The **conversion value** of a convertible bond is the value of the common stock into which the bond can be converted. The conversion ratio is the number of shares the holder receives from conversion for each bond. Conversion value is calculated as:

conversion value = market price of stock × conversion ratio

The **straight value**, or investment value, of a convertible bond is the value of the bond if it were not convertible—the present value of the bond's cash flows discounted at the return required on a comparable option-free issue.

The **minimum value of a convertible bond** is the greater of its conversion value or its straight value. This must be the case, or arbitrage opportunities would be possible. For example, if a convertible bond were to sell for less than its conversion value, it could be purchased, immediately converted into common stock, and the stock could be sold for more than the cost of the bond.

minimum value of a convertible bond = max (straight value, conversion value)

EXAMPLE: Calculating the minimum value of a convertible bond

Business Supply Company, Inc. operates retail office equipment stores in the United States and Canada. Consider a BSC convertible bond with a 7% coupon that is currently selling at $985 with a conversion ratio of 25 and a straight value of $950. Suppose that the value of BSC's common stock is currently $35 per share, and that it pays $1 per share in dividends annually. What is this bond's minimum value?

Answer:

The conversion value of this bond is 25 × $35 = $875. Since the straight value of $950 is greater than the conversion value of $875, the bond is worth at least $950.

The market conversion price, or conversion parity price, is the price that the convertible bondholder would effectively pay for the stock if she bought the bond and immediately converted it. The market conversion price is given as:

$$\text{market conversion price} = \frac{\text{market price of convertible bond}}{\text{conversion ratio}}$$

EXAMPLE: Calculating market conversion price

Compute and interpret the market conversion price of the BSC bond.

Answer:

The market conversion price is: $985 / 25 = $39.40. This can be viewed as the stock price at which an investor is indifferent between selling the bond and converting it.

The market conversion premium per share is the difference between the market conversion price and the stock's current market price:

$$\text{market conversion premium per share} = \text{market conversion price} - \text{stock's market price}$$

EXAMPLE: Calculating market conversion premium per share

Compute and interpret the market conversion premium per share of the BSC bond.

Answer:

Since BSC is selling for $35 per share, the market conversion premium per share for the BSC bond is: $39.40 – $35 = $4.40. This can be interpreted as the premium that investors are willing to pay for the opportunity to profit should the market price of the stock rise above the market conversion price. This is done with the assurance that even if the stock price declines, the value of the convertible bond will not fall below its straight value.

Market conversion premium per share is usually expressed as a ratio, appropriately called the **market conversion premium ratio**. Its formula is:

$$\text{market conversion premium ratio} = \frac{\text{market conversion premium per share}}{\text{market price of common stock}}$$

EXAMPLE: Calculating market conversion premium ratio

Compute the market conversion premium ratio of the BSC bond.

Answer:

The BSC bond market conversion premium ratio is:

$$\frac{\$4.40}{\$35} = 12.57\%$$

The convertible bond investor's downside risk is limited by the bond's underlying straight value because the price of a convertible bond will not fall below this value regardless of what happens to the price of the issuer's common stock.

This downside risk is measured by the **premium over straight value**, which is calculated as:

$$\text{premium over straight value} = \left(\frac{\text{market price of convertible bond}}{\text{straight value}}\right) - 1$$

EXAMPLE: Calculating premium over straight value

Compute and interpret the BSC bond's premium over straight value.

Answer:

The premium over straight value for the BSC bond is:

$$\left(\frac{\$985}{\$950}\right) - 1 = 3.68\%$$

All else equal, the greater the premium over straight value, the less attractive the convertible bond.

We need to recognize an obvious flaw with the premium over straight value metric —the straight value is not constant. It varies with changes in interest rate and with the credit spread of the bond.

LOS 27.p: Describe how a convertible bond is valued in an arbitrage-free framework.

Investing in a noncallable/nonputable convertible bond is equivalent to buying:

- an option-free bond, and
- a call option on an amount of the common stock equal to the conversion ratio.

The value of a noncallable/nonputable convertible bond can be expressed as:

$$\begin{aligned}\text{convertible, noncallable bond value} = &\ \text{straight value}\\ &+ \text{value of call option on stock}\end{aligned}$$

Most convertible bonds are callable, giving the issuer the right to call the issue prior to maturity. Incorporating this feature into the valuation of a convertible bond results in the following expression:

$$\begin{aligned}\text{callable convertible bond value} = &\ \text{straight value of bond}\\ &+ \text{value of call option on stock}\\ &- \text{value of call option on bond}\end{aligned}$$

To further complicate the situation (just for fun), consider a convertible bond that is both callable and putable. The expression for value then becomes:

$$\begin{aligned}\text{callable and putable convertible bond value} = &\ \text{straight value of bond}\\ &+ \text{value of call option on stock}\\ &- \text{value of call option on bond}\\ &+ \text{value of put option on bond}\end{aligned}$$

LOS 27.q: Compare the risk–return characteristics of a convertible bond with the risk–return characteristics of a straight bond and of the underlying common stock.

Buying convertible bonds instead of stocks limits downside risk; the price floor set by the straight bond value provides this downside protection. The cost of the downside protection is reduced upside potential due to the conversion premium. Keep in mind though, that just like investing in nonconvertible bonds, convertible bond investors must be concerned with credit risk, call risk, interest rate risk, and liquidity risk.

Consider the following two examples based on our previous BSC example.

EXAMPLE: Risk and return of a convertible bond, part 1

Calculate the return on the convertible bond and the common stock if the market price of BSC common stock increases to $45 per share.

Answer:

The return from investing in the convertible bond is:

$$\left(\frac{\$45.00}{\$39.40}\right) - 1 = 14.2\%$$

The return from investing directly in the stock is:

$$\left(\frac{\$45.00}{\$35.00}\right) - 1 = 0.2857 = 28.6\%$$

The lower return from the convertible bond investment is attributable to the fact that the investor effectively bought the stock at the market conversion price of $39.40 per share.

EXAMPLE: Risk and return of a convertible bond, part 2

Calculate the return on the convertible bond and on the common stock if the market price of BSC common stock falls to $30 per share.

Answer:

Recall that the bond will trade at the greater of its straight value or its conversion value. The conversion value in this scenario is 25 × $30 = $750. Assuming the straight value of the bond does not change, the bond will trade at $950. So, the return from investing in the convertible bond is:

$$\left(\frac{\$950}{\$985}\right) - 1 = -3.55\%$$

The return from investing directly in the stock is:

$$\left(\frac{\$30}{\$35}\right) - 1 = -14.29\%$$

The loss is less for the convertible bond investment because we assumed that the straight value of the bond did not change. Even if it had changed, the loss would probably still be less than the loss on the straight stock investment, thus emphasizing how the straight value serves as a floor to cushion a decline, even if it is a moving floor.

The following comparisons can be made between ownership of the underlying stock and the risk-return characteristics of the convertible bond:

- When the stock's price falls, the returns on convertible bonds exceed those of the stock, because the convertible bond's price has a floor equal to its straight bond value. As the stock's price approaches zero, the convertible's value will move toward the present value of estimated recovery.

- When the stock's price rises, the bond will underperform because of the conversion premium. This is the main drawback of investing in convertible bonds versus investing directly in the stock.
- If the stock's price remains stable, the return on a convertible bond may exceed the stock return due to the coupon payments received from the bond, assuming no change in interest rates or credit risk of the issuer.

Sometimes the price of the common stock associated with a convertible issue is so low that it has little or no effect on the convertible's market price, and the bond trades as though it is a straight bond. When this happens, the convertible security is referred to as a fixed-income equivalent or a **busted convertible**.

Other times, the price of the stock may be high enough that the convertible behaves as though it is an equity security. When this happens, the convertible issue is referred to as a common stock equivalent. Most of the time, however, the convertible behaves as a *hybrid security* with the characteristics of both equity and a fixed-income security.

MODULE QUIZ 27.8

1. An analyst has gathered the following information on a convertible bond and the common equity of the issuer.
   - Market price of bond: $925.00
   - Annual coupon: 7.5%
   - Conversion ratio: 30
   - Market price of stock: $28.50
   - Annual stock dividend: $2.15 per share

   The market conversion premium ratio for the convertible bond is *closest* to:
   A. 7.56%.
   B. 7.77%.
   C. 8.18%.
2. Which of the following statements concerning a comparison between the risk and return of convertible bond investing versus common stock investing is *least accurate*, assuming interest rates are stable?
   A. When stock prices fall, the returns on convertible bonds are likely to exceed those of the stock because the convertible bond's price has a floor equal to the straight bond value.
   B. The main drawback of investing in convertible bonds versus direct stock purchases is that when stock prices rise, the convertible bond will likely underperform the stock due to the conversion premium.
   C. Buying convertible bonds instead of direct stock investing limits upside potential to that of buying a straight bond, at the cost of increased downside risk due to the conversion premium.
3. Data on two convertible bonds are shown in the following table.

	Convertible Bond ABC	Convertible Bond XYZ
Conversion price	$40	$50
Current stock price	$123	$8

   Which factors are *most likely* to influence the market prices of ABC and XYZ: factors that affect equity prices, or factors that affect option-free bond prices?
   A. Both will be more affected by equity factors.

B. One will be more affected by equity factors, the other by bond factors.
C. Both will be more affected by bond factors.

4. The difference between the value of a callable convertible bond and the value of an otherwise comparable option-free bond is *closest* to the value of the:
   A. call option on the stock minus value of the call option on the bond.
   B. put option on the stock plus value of the call option on the bond.
   C. call option on the stock plus value of call option on the bond.

5. With respect to the value of a callable convertible bond, the *most likely* effects of a decrease in interest rate volatility or a decrease in the underlying stock price volatility are:
   A. both will result in an increase in value.
   B. one will result in an increase in value, the other in a decrease.
   C. both will result in a decrease in value.

KEY CONCEPTS

LOS 27.a

Bonds with embedded options allow issuers to manage their interest rate risk or issue bonds at attractive coupon rates. The embedded options can be simple call or put options, or more complex options such as provisions for sinking fund, estate puts, et cetera.

LOS 27.b

Value of option embedded in a callable or putable bond:

$$V_{call} = V_{straight} - V_{callable}$$

$$V_{put} = V_{putable} - V_{straight}$$

LOS 27.c

To value a callable or a putable bond, the backward induction process and a binomial interest rate tree framework is used. The benchmark binomial interest rate tree is calibrated to ensure that it values benchmark bonds correctly (i.e., that it generates prices equal to their market prices).

LOS 27.d

When interest rate volatility increases, the value of both call and put options on bonds increase. As volatility increases, the value of a callable bond decreases (remember that the investor is short the call option) and the value of a putable bond increases (remember that the investor is long the put option).

LOS 27.e

The short call in a callable bond limits the investor's upside when rates decrease, while the long put in a putable bond hedges the investor against rate increases.

The value of the call option will be lower in an environment with an upward-sloping yield curve as the probability of the option going in the money is low. A call option gains value when the upward-sloping yield curve flattens. A put option will have a higher probability of going in the money when the yield curve is upward sloping; the option loses value if the upward-sloping yield curve flattens.

LOS 27.f

A backwards induction process is used in a binomial interest rate tree framework for valuing a callable (or putable) bond. In the binomial tree, we use one-period forward rates for each period. For valuing a callable (putable) bond, the value used at any node corresponding to a call (put) date must be either the price at which the issuer will call (investor will put) the bond, or the computed value if the bond is not called (put)—whichever is lower (higher).

LOS 27.g

The option adjusted spread (OAS) is the constant spread added to each forward rate in a benchmark binomial interest rate tree, such that the sum of the present values of a credit risky bond's cash flows equals its market price.

LOS 27.h

Binomial trees generated under an assumption of high volatility will lead to higher values for a call option and a corresponding lower value for a callable bond. Under a high volatility assumption, we would already have a lower computed value for the callable bond, and hence, the additional spread (i.e., the OAS) needed to force the discounted value to equal the market price will be lower.

When an analyst uses a lower-than-actual (higher-than-actual) level of volatility, the computed OAS for a callable bond will be too high (low) and the bond will be erroneously classified as underpriced (overpriced).

Similarly, when the analyst uses a lower-than-actual (higher-than-actual) level of volatility, the computed OAS for a putable bond will be too low (high) and the bond will be erroneously classified as overpriced (underpriced).

LOS 27.i

$$\text{effective duration} = \text{ED} = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$$

LOS 27.j

effective duration (callable) ≤ effective duration (straight)

effective duration (putable) ≤ effective duration (straight)

effective duration (zero-coupon) ≈ maturity of the bond

effective duration of fixed-rate coupon bond < maturity of the bond

effective duration of floater ≈ time (in years) to next reset

LOS 27.k

For bonds with embedded options, one-sided durations—durations when interest rates rise versus when they fall—are better at capturing interest rate sensitivity than the more common effective duration. When the underlying option is at (or near) the money, callable (putable) bonds will have lower (higher) one-sided down-duration than one-sided up-duration.

Callable bonds with low coupon rates will most likely not be called and hence their maturity matched rate is their most critical rate (and has the highest key rate

duration). As the coupon rate increases, a callable bond is more likely to be called and the time-to-exercise rate will start dominating the time-to-maturity rate.

Putable bonds with high coupon rates are unlikely to be put and are most sensitive to its maturity-matched rate. As the coupon rate decreases, a putable bond is more likely to be put and the time-to-exercise rate will start dominating the time-to-maturity rate.

LOS 27.l

Straight and putable bonds exhibit positive convexity throughout. Callable bonds also exhibit positive convexity when rates are high. However, at lower rates, callable bonds exhibit negative convexity.

LOS 27.m

A capped floater contains an issuer option that prevents the coupon rate on a floater from rising above a specified maximum (i.e., the cap) rate.

value of a capped floater
= value of a "straight" floater − value of the embedded cap

A related floating-rate bond is the floored floater where the coupon rate will not fall below a specified minimum (i.e., the floor).

value of a floored floater
= value of a "straight" floater + value of the embedded floor

LOS 27.n

The owner of a convertible bond can exchange the bond for the common shares of the issuer. A convertible bond includes an embedded call option giving the bondholder the right to buy the common stock of the issuer.

LOS 27.o

The conversion ratio is the number of common shares for which a convertible bond can be exchanged.

conversion value = market price of stock × conversion ratio

market conversion price = market price of convertible bond / conversion ratio

market conversion premium per share = market conversion price − market price

The minimum value at which a convertible bond trades is its straight value or its conversion value, whichever is greater.

LOS 27.p

The value of a bond with embedded options is determined as the value of the straight bond plus (minus) the value of options that the investor is long (short).

callable and putable convertible bond value = straight value of bond
+ value of call option on stock
− value of call option on bond
+ value of put option on bond

LOS 27.q

- The major benefit from investing in convertible bonds is the price appreciation resulting from an increase in the value of the common stock.

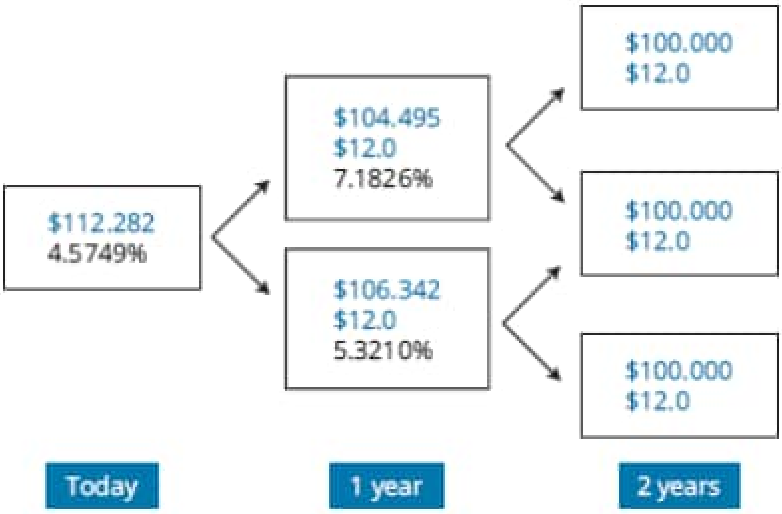
- The main drawback of investing in a convertible bond versus investing directly in the stock is that when the stock price rises, the bond will underperform the stock because of the conversion premium of the bond.
- If the stock price remains stable, the return on the bond may exceed the stock returns due to the coupon payments received from the bond.
- If the stock price falls, the straight value of the bond limits downside risk (assuming bond yields remain stable).

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 27.1, 27.2

1. **B** The value at any given node in a binomial tree is the average of the present values of the cash flows at the two possible states immediately to the right of the given node, discounted at the one-period rate at the node under examination. (Module 27.2, LOS 27.c)

2. **C** The tree should look like this:



Consider the value of the bond at the *upper* node for Period 1, $V_{1,U}$:

$$V_{1,U} = \frac{1}{2} \times \left[\frac{\$100 + \$12}{1.071826} + \frac{\$100 + \$12}{1.071826}\right] = \$104.495$$

Similarly, the value of the bond at the *lower* node for Period 1, $V_{1,L}$ is:

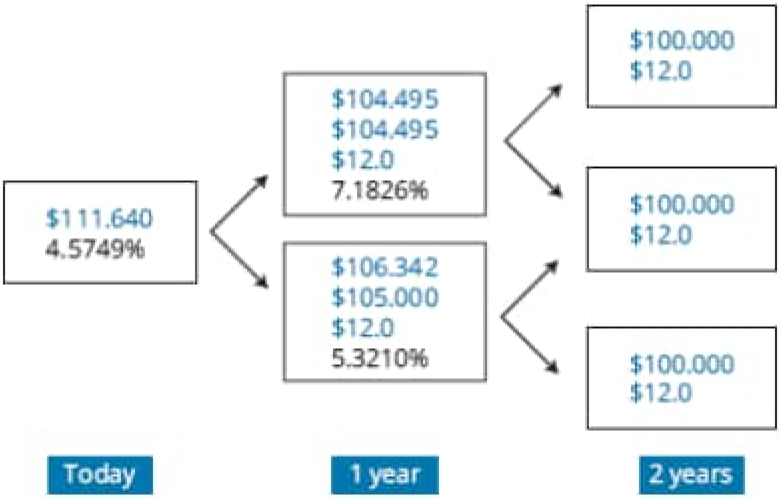
$$V_{1,L} = \frac{1}{2} \times \left[\frac{\$100 + \$12}{1.053210} + \frac{\$100 + \$12}{1.053210}\right] = \$106.342$$

Now calculate V_0, the current value of the bond at Node 0:

$$V_0 = \frac{1}{2} \times \left[\frac{\$104.495 + \$12}{1.045749} + \frac{\$106.342 + \$12}{1.045749}\right] = \$112.282$$

(Module 27.2, LOS 27.f)

3. **C** The tree should look like this:



Consider the value of the bond at the *upper* node for Period 1, $V_{1,U}$:

$$V_{1,U} = \frac{1}{2} \times \left[\frac{\$100 + \$12}{1.071826} + \frac{\$100 + \$12}{1.071826}\right] = \$104.495$$

Similarly, the value of the bond at the *lower* node for Period 1, $V_{1,L}$ is:

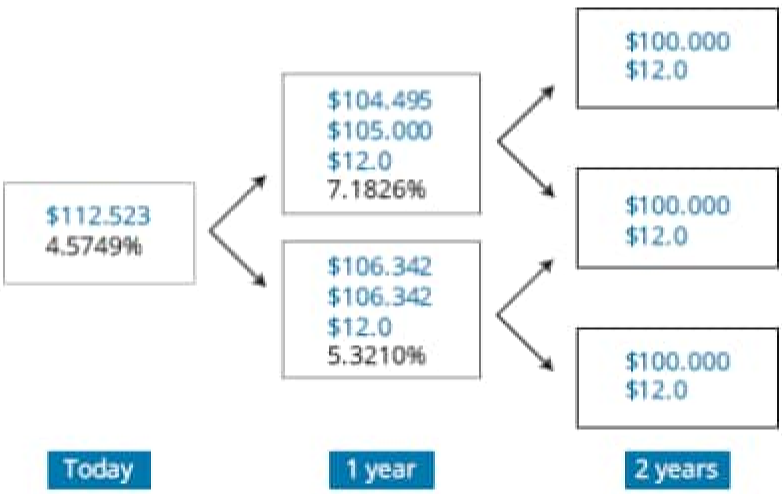
$$V_{1,L} = \frac{1}{2} \times \left[\frac{\$100 + \$12}{1.053210} + \frac{\$100 + \$12}{1.053210}\right] = \$106.342$$

Now calculate V_0, the current value of the bond at Node 0:

$$V_0 = \frac{1}{2} \times \left[\frac{\$104.495 + \$12}{1.045749} + \frac{\$105.00 + \$12}{1.045749}\right] = \$111.640$$

The value of the embedded call option is $112.282 – $111.640 = $0.642. (Module 27.2, LOS 27.f)

4. **A** The tree should look like this:



Consider the value of the bond at the *upper* node for Period 1, $V_{1,U}$:

$$V_{1,U} = \frac{1}{2} \times \left[\frac{\$100 + \$12}{1.071826} + \frac{\$100 + \$12}{1.071826}\right] = \$104.495$$

Similarly, the value of the bond at the *lower* node for Period 1, $V_{1,L}$, is:

$$V_{1,L} = \frac{1}{2} \times \left[\frac{\$100 + \$12}{1.053210} + \frac{\$100 + \$12}{1.053210}\right] = \$106.342$$

Now calculate V_0, the current value of the bond at Node 0:

$$V_0 = \frac{1}{2} \times \left[\frac{\$105.000 + \$12}{1.045749} + \frac{\$106.342 + \$12}{1.045749}\right] = \$112.523$$

The value of the embedded put option is \$112.523 – \$112.282 = \$0.241. (Module 27.2, LOS 27.f)

Module Quiz 27.3, 27.4

1. **C** Let's construct a table of the risk differences between the issuer's callable bond and on-the-run Treasuries to help us answer this question.

Type of Risk	Equal?
Credit	No
Option	Removed by OAS

Therefore, the OAS reflects the credit risk of the corporate callable bond over Treasuries since option risk has been removed. (Module 27.4, LOS 27.g)

2. **B** Like ordinary options, the value of an embedded option increases as volatility increases. Furthermore, the arbitrage-free value of an option-free bond ($V_{option\text{-}free}$) is independent of the assumed volatility. This implies that the arbitrage-free value of a callable bond ($V_{callable}$) decreases as volatility increases the value of the embedded call option (V_{call}). This can be seen from the expression for the value of a callable bond:

$$\downarrow V_{callable} = V_{option\text{-}free} - \uparrow V_{call}$$

The value of the putable bond ($V_{putable}$) increases as the assumed volatility increases the value of the embedded put option (V_{put}).

$$\uparrow V_{putable} = V_{option\text{-}free} + \uparrow V_{put}$$

(Module 27.3, LOS 27.d)

Module Quiz 27.5

1. **B** The duration formula is presented correctly. The convexity formula is presented incorrectly; the "2" should not appear in the denominator of the convexity formula.

$$\text{effective convexity} = EC = \frac{BV_{-\Delta y} + BV_{+\Delta y} - (2 \times BV_0)}{BV_0 \times \Delta y^2}$$

(LOS 27.i)

Module Quiz 27.6, 27.7

1. **A** When an upward sloping yield curve flattens, call options increase in value while put options decrease in value. (Module 27.3, LOS 27.e)

2. **A** When the assumed volatility in a binomial tree increases, the computed value of OAS will decrease. When an analyst uses a lower-than-actual level of volatility like the 15% volatility assumed here, the computed OAS for a callable bond will be too high. Using the 19% implied volatility instead would have resulted in an estimated OAS lower than 145 bps. (Module 27.4, LOS 27.h)

3. **A** Straight bonds generally have higher effective durations than bonds with embedded options. Both call and put options have the potential to reduce the life of a bond, so the duration of callable and putable bonds will be less than or equal to that of their straight counterparts. (Module 27.5, LOS 27.j)

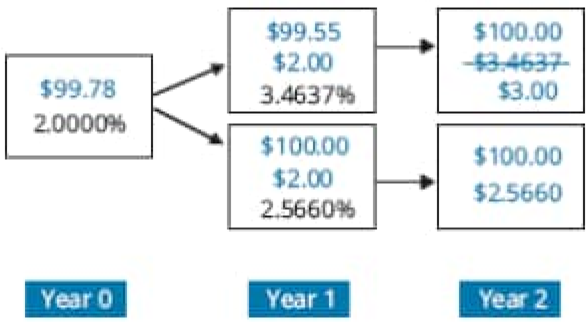
4. **A** Statement 1 is incorrect. Low-coupon callable bonds are unlikely to be called; hence, their highest key rate duration corresponds to their time-to-maturity. Statement 2 is correct. High coupon putable bonds are unlikely to be put and hence, their highest key rate duration corresponds to their time-to-maturity. (Module 27.6, LOS 27.k)

5. **B** Straight and putable bonds exhibit positive convexity at all interest rate levels. The price appreciation for a callable bond due to decline in interest rate is limited due to the call feature; callables exhibit negative convexity at low rates. Hence a decline in rates is least likely to result in best price performance for a callable bond. (Module 27.6, LOS 27.l)

6. **B** When interest rates increase, a callable bond becomes less likely to be called (its duration will increase). The put option in a putable bond would be more likely to be exercised in a rising interest rate scenario and hence, the duration of a putable bond would decrease. Duration of an option-free bond would also decrease as interest rate increases (but not as significantly). (Module 27.6, LOS 27.k)

7. **A** Relative to bond A, bond B has lower OAS. Given that the two bonds have similar credit risk, bond B offers a lower OAS for the same level of risk as bond A and thus would be considered overpriced. Alternatively, bond A is more attractive (underpriced) relative to bond B. (Module 27.4, LOS 27.g)

8. **C** The value of the capped floater is $99.78, as shown here:



The upper node at Year 2 is subject to the 3% cap, and the coupon is adjusted accordingly.

$V_{1,U} = \$103 / (1.03467) = \$99.55$

$V_{1,L} = 100$ *(option is not exercised)*

$$V_0 = \frac{[(100 + 2) + (99.55 + 2)]/2}{(1.02)} = 99.78$$

(Module 27.7, LOS 27.m)

Module Quiz 27.8

1. **C** The market conversion premium per share is the market conversion price per share minus the market price per share. The market conversion price per share is $\frac{925.00}{30} = \$30.833$, so the conversion premium per share is \$30.833 – \$28.50 = \$2.333.

$$\text{market conversion premium ratio} = \frac{\text{market conversion premium per share}}{\text{market price of common stock}} = \frac{2.33}{28.50} = 8.18\%$$

(LOS 27.o)

2. **C** Buying convertible bonds in lieu of direct stock investing limits downside risk to that of straight bond investing, at the cost of reduced upside potential due to the conversion premium. (Note that this analysis assumes that interest rates remain stable. Otherwise, the interest rate risk associated with the straight bond investing must be considered.) When stock prices fall, the returns on convertible bonds are likely to exceed those of the stock, because the convertible bond's price has a floor equal to the straight bond value. The main drawback of investing in convertible bonds versus direct stock purchases is that when stock prices rise, the convertible bond is likely to underperform due to the conversion premium. If the stock price remains stable, the return on the bond may exceed the stock's return if the bond's coupon payment exceeds the dividend income of the stock. (LOS 27.q)

3. **B** ABC has a conversion price much less than the current stock price, so the conversion option is deep in the money. Bond ABC effectively trades like equity and is more likely to be influenced by the same factors that affect equity prices, in general, than the factors that affect bond prices.

   A busted convertible like XYZ, with a stock price significantly less than the conversion price, trades like a bond (that's why a busted convertible is also called a fixed-income equivalent) and is therefore more likely to be influenced by the factors that affect bond prices. (LOS 27.q)

4. **A** A bond that is both callable and convertible contains two embedded options: (1) a call option on the stock and (2) a call option on the bond. The investor has a *short* position in the call option on the bond (the issuer has the right to call the bond) and a *long* position in the call option on the stock (the investor has the right to convert the bond into shares of stock). Therefore, the difference in value between the callable convertible bond and the value of the comparable option-free bond to the investor is equal to the value of the call option on the stock minus the value of the call option on the bond. (LOS 27.p)

5. **B** A decrease in interest rate volatility will decrease the value of the embedded short call on the bond (but have no effect on the value of the embedded call on the stock) and increase the value of the convertible bond.

   A decrease in stock price volatility will decrease the value of the embedded call on the stock (but have no effect on the embedded call on the bond) and decrease the value of the convertible bond. (LOS 27.p)

READING 28

CREDIT ANALYSIS MODELS

EXAM FOCUS

This topic review augments credit analysis covered at Level I with newer models of credit analysis including structural models and reduced form models. Be able to differentiate between the two and know the merits and drawbacks of both. Be able to compute the credit valuation adjustment and expected change in a bond's price given credit migration, and credit spread given CVA. Finally, understand how credit analysis of ABS differs from credit analysis of corporate debt.

This topic review provides a broad overview of evaluation of credit risk. Do note that while credit risk and default risk are often used interchangeably, credit risk is a broader term. Credit risk *includes* the risk of default, as well as the risk of worsening credit quality (even if default does not occur).

MODULE 28.1: CREDIT RISK MEASURES

Video covering this content is available online.

LOS 28.a: Explain expected exposure, the loss given default, the probability of default, and the credit valuation adjustment.

Expected exposure is the amount of money a bond investor in a credit risky bond stands to lose at a point in time *before* any recovery is factored in. The expected exposure is equal to the present value (based on the risk-free rate) of the bond's remaining cash flows. A bond's expected exposure changes over time. **Recovery rate** is the percentage recovered in the event of a default. Recovery rate is the opposite of loss severity; given a loss severity of 40%, the recovery rate would be 60%. **Loss given default (LGD)** is equal to loss severity multiplied by exposure.

Probability of default is the likelihood of default occurring in a given year. The **hazard rate** is the conditional probability of default given that default has previously not occurred. **Probability of survival** is 1 – the cumulative probability of default. If we assume a constant hazard rate, then the probability of survival for time period t is given as:

$$PS_t = (1 - \text{hazard rate})^t$$

From this expression, it can be seen that the probability of survival decreases over time. PD, the probability of default, depends on the probability of survival from the prior period.

$$PD_t = \text{hazard rate} \times PS_{(t-1)}$$

In the first year, the probability of default = hazard rate (because PS = 1 at inception). In subsequent years, PD < hazard rate. The **expected loss** for any period is the loss given default (LGD) for that period multiplied by the PD for that period.

Credit valuation adjustment (CVA) is the sum of the present value of the expected loss for each period. CVA is the monetary value of the credit risk in present value terms; it is the difference in value between a risk-free bond and an otherwise identical risky bond:

CVA = price of risk-free bond – price of risky bond.

We illustrate the computation of CVA in the following example.

EXAMPLE: Credit Valuation Adjustment (CVA), Part 1

A 3-year, $100 par, zero-coupon corporate bond has a hazard rate of 2% per year. Its recovery rate is 60% and the benchmark rate curve is flat at 3%. Calculate the expected exposure, probability of survival, probability of default, loss given default, CVA, and the credit spread on the bond.

Answer:

Year	Exposure	LGD	PS	PD	Expected Loss	DF	PV of Expected Loss
1	94.260	37.704	98.000%	2.000%	0.754077	0.970874	0.732113
2	97.087	38.835	96.040%	1.9600%	0.761165	0.942596	0.717471
3	100.000	40.000	94.119%	1.9208%	0.76832	0.915142	0.703122
						CVA	2.152706

Exposure at the end of three years is the face value of the bond ($100). Exposure at the end of Year 2 is the present value of $100 discounted for 1 period at 3% (= $100 / 1.03). Exposure at the end of Year 1 is the present value discounted for 2 periods (= $100 / 1.03^2).

Loss given default (LGD) is (1 – recovery rate) or 40% of the exposure.

Probabilities of survival (PS) for Years 1, 2, and 3 are calculated as $(1 - 0.02)^1$ = 0.98, $(1 - 0.02)^2$ = 0.9604, and $(1 - 0.02)^3$ = 0.9412.

Probability of default is 2% in the first year, and decreases with the PS. PD for Year 2 = 2% of PS for Year 1 = 0.02 × 0.98 = 0.0196 or 1.96%. Similarly, the PD for Year 3 = 0.02 × 0.9604 = 0.019208 or 1.9208%.

expected loss for each year = LGD × PD.

expected loss for Year 3 = 0.019208 × 40 = 0.7683.

DF is the present value of $1. Year 2 DF = 1 / $(1.03)^2$ = 0.942596.

PV of expected loss = DF × expected loss

CVA = sum of the three PV of expected loss = $2.15

An identical benchmark bond should trade at $100 / 1.03^3 = $91.51. Therefore, the value of credit risky bond = 91.50 – CVA = 91.51 – 2.15 = $89.36.

Credit spread calculation:

Step 1: YTM on risk-free bond (given): 3%

Step 2: YTM on risky bond:

N = 3, PV = –89.36, PMT = 0, FV = 100, CPT I/Y = 3.82%

Step 3: Credit spread = YTM risky – YTM risk-free = 3.82% – 3% = 0.82%

EXAMPLE: Credit Valuation Adjustment (CVA), Part 2

Continuing the previous example: now suppose that the bond pays a 4% annual coupon and everything else is the same.

Calculate the expected exposure, probability of survival, probability of default, loss given default, and CVA.

Answer:

Year	Exposure	LGD	PS	PD	Expected Loss	DF	PV of Expected Loss
1	105.913	42.365	98.000%	2.000%	0.847308	0.970874	0.822629
2	104.971	41.988	96.040%	1.960%	0.822972	0.942596	0.775730
3	104.000	41.600	94.119%	1.921%	0.799053	0.915142	0.731247
						CVA	2.329605

The probability of default and discount factors will remain the same as before, but the expected exposure changes due to the coupon.

Exposure at the end of three years is the face value of the bond plus the final coupon ($104). Exposure at the end of Year 2 is the present value of $104 discounted at 3% for 1 period ($104 / 1.03 = $100.971), plus the $4 Year 2 coupon ($100.971 + $4.00 = $104.971). Exposure at the end of Year 1 is the present value of the Year 3 cash flows discounted for 2 periods, plus the present value of the Year 2 coupon, plus the Year 1 coupon: ($\$104/1.03^2 + 4/1.03 + 4 = \$105.913$).

Other calculations follow the same process.

Risk Neutral Probability of Default

In the previous example, we used a probability of default that was based on the expected likelihood of default in any given year. However, in practice, we use the *risk neutral probability* of default, which is the probability of default *implied* in the current market price.

While the computation of risk-neutral probability of default is not needed for the exam, consider a 1-year, zero-coupon, $100 par bond trading at $95. The benchmark 1-year rate is 3% and the recovery rate is assumed to be 60%. At the end of the year, there are two possible outcomes: either the bond does not default (pays the promised par value of $100) or defaults (the cash flow is recovery rate × 100 = $60).

Assuming a probability of default of p, the expected year-end cash flow = $60p + 100(1 - p) = 100 - 40p$. Using the risk-free (benchmark) rate, the present value of this expected cash flow = $100 - 40p / (1.03)$.

Setting this value to be equal to the bond's market price:

$$95 = \frac{100 - 40p}{1.03}$$

implies $p = 5.38\%$.

While calculating the risk neutral probability of default, we assumed a recovery rate of 60%. Using similar mechanics, we can also calculate the implied recovery rate in the market price *given* the probability of default. If we had assumed a probability of default greater than 5.38%, then the implied recovery rate would have been higher (keeping the market price constant).

In general, given the market price (and hence the credit spread), the estimated risk neutral probabilities of default and recovery rates are positively correlated.

PROFESSOR'S NOTE

This is an important statement. We need to recognize that there are several inputs in the bond valuation model—some are known (such as the bond's coupon rate, maturity, and the benchmark term structure) while two of the inputs are estimates (probability of default and recovery rate). We have to assume one to calculate the value of the other implied in the current market price.

ESG Considerations

Analysts should also consider environmental, social, and governance factors when evaluating the default risk of the company. For example, polluters may violate environmental regulations, resulting in fines or business curtailments. Companies with poor labor practices may experience loss of reputation, customers, and profitability. Similarly, companies with poor governance systems may resort to fraudulent accounting to mask debt service problems. On the flip side, some countries provide tax incentives for investing in **green bonds** (i.e., bonds that fund green projects). The WHO issued pandemic bonds in 2017 that offered a high interest rate, but in the event of a pandemic, the principal would be diverted as aid to poor countries. Due to the COVID-19 pandemic, by July 2020, the entire principal of these bonds was wiped out.

MODULE QUIZ 28.1

1. Manny Zeld is evaluating a 5%, annual pay, $100 par, 5-year Barry Corp. bond and has calculated the CVA as $12.67. Benchmark rates are flat at 2%. Zeld would *most appropriately* value Barry Corp. bonds at:
   A. $101.47.
   B. $110.22.
   C. $114.76.
2. If the annual hazard rate for a bond is 1.25%, the probability of default (PD) in Year 1 is:
   A. less than 1.25%.
   B. equal to 1.25%.
   C. greater than 1.25%.
3. If the annual hazard rate for a bond is 1.25%, the probability that the bond does not default over the next three years is *closest* to:
   A. 94.32%.
   B. 95.20%.
   C. 96.30%.

4. For a risky bond, the expected loss for a specific year is *most appropriately* calculated as:
   A. exposure multiplied by the recovery rate multiplied by the hazard rate.
   B. loss given default multiplied by the probability of default.
   C. exposure multiplied by the probability of default.
5. The CVA for a risky bond is *most likely* the:
   A. price discount for a credit risky bond compared to an otherwise identical risk-free bond.
   B. cumulative amount of expected loss.
   C. mean cumulative present value of expected loss.
6. Joel Abramson, CFA, is the fixed income portfolio manager of VZ Bank. Based on his analysis of bonds issued by Tinta Corp, including an evaluation of Tinta's balance sheet, Joel estimates the hazard rate for the bond. Abramson would *most accurately* consider the bond to be an attractive purchase if the:
   A. estimated hazard rate is less than the bond's risk-neutral probability of default.
   B. estimated hazard rate is less than 2%.
   C. estimated hazard rate is greater than the risk-neutral probability of default.
7. Given a risky bond's market price, its risk-neutral probability of default is *most likely* to be:
   A. positively correlated with the assumed recovery rate.
   B. negatively correlated with the assumed recovery rate.
   C. independent of the assumed recovery rate.

MODULE 28.2: ANALYSIS OF CREDIT RISK

Video covering this content is available online.

Consider our previous example where we were evaluating a 3-year, \$100 par, zero-coupon corporate bond. Its recovery rate is 60% and benchmark rates are flat at 3%. We calculated the exposures at the end of Years 1, 2, and 3 as \$94.26, \$97.09, and \$100, respectively. CVA was calculated as \$2.15. The bond price today = value of risk-free bond – CVA = $[100 / (1.03)^3] - 2.15$ = \$89.36. Assume that default only occurs at year-end.

The cash flows on this bond in the event of default and corresponding internal rate of return (IRR), as well as the IRR if the bond does not default, are shown in Figure 28.1.

Figure 28.1: Expected Rates of Return

	Year 1		Year 2		Year 3	
	Cash Flow	IRR	Cash Flow	IRR	Cash Flow	IRR
Default	\$56.56	–36.71%	\$58.25	–19.26%	\$60.00	–12.43%
No Default	0	–	0	–	\$100.00	3.82%

At the end of Year 1, there are two possible outcomes: default or no-default. If default occurs, a recovery of 60% of exposure or 0.6 × 94.26 = \$56.56 occurs. Similarly, in Years 2 and 3, in case of default, the recovery amounts are \$58.25 and \$60, respectively.

If the bond defaults in Year 1, the investor's IRR is calculated as:

PV = –89.36, N = 1, FV = 56.56, CPT I/Y = –36.71%

Similarly, the IRR in case default occurs in Years 2 and 3 are:

Year 2: PV = –89.36, N = 2, FV = 58.25, CPT I/Y = –19.26%

Year 3: PV = –89.36, N = 3, FV = 60.0, CPT I/Y = –12.43%

If the bond does not default over its life, the investor would earn an IRR of:

PV = –89.36, N = 3, FV = 100.0, CPT I/Y = 3.82%

Relative Credit Risk Analysis

While comparing the credit risk of several bonds, the metric that combines the probability of default as well as loss severity is the **expected loss**. Everything else constant, for a given period, the higher the expected loss, the higher the credit risk.

EXAMPLE: Relative risk evaluation

Elsa Jaitley is comparing three corporate bonds for inclusion in her fixed-income portfolio. For the next year, Jaitley has collected the following information on the three bonds:

Bond	Exposure (per $100 par)	Recovery (per $100 par)	Probability of Default
X	102	40	1.25%
Y	88	45	1.30%
Z	92	32	1.65%

On a relative basis, which bond has the highest risk? Based on this information, what would be an appropriate trading strategy?

Answer:

Credit risk can be evaluated based on the expected loss.

Bond	Exposure (per $100 par)	Recovery (per $100 par)	Probability of Default	LGD (per $100 par)	Expected Loss
X	102	40	1.25%	62	0.775
Y	88	45	1.30%	43	0.559
Z	92	32	1.65%	60	0.99

Since the exposure and recovery amounts are given per $100 par,

loss given default (LGD) per $100 par = exposure – recovery

For example, for bond Y, LGD = 88 – 45 = $43.

expected loss = LGD × probability of default

Based on expected loss, bond Z is the most risky while bond Y has the least credit risk.

Because we are not given the market price of the three bonds, no trading strategy can be recommended based on only this information.

MODULE QUIZ 28.2

1. A 3-year, zero-coupon corporate bond trades at a price of $89.49. The benchmark yield curve is flat at 3%. Given a recovery rate of 45%, what is the IRR on the bond if it defaults in Year 2?

A. –24.66%.
B. –27.43%.
C. –30.13%.

2. Given the following information, which bond has the *least* amount of credit risk?

Bond	Exposure (per $100 par)	Recovery (per $100 par)	Probability of Default
P	98	50	2.50%
Q	94	56	3.00%
R	89	49	4.65%

A. Bond P.
B. Bond Q.
C. Bond R.

3. A bond trader observes a 5% annual-pay, 3-year, corporate bond trading at $103. Benchmark rates are flat at 2.50%. The trader has collected the following information on the bond:

Year	Exposure	LGD	PD	PS	Expected Loss	DF	PV of Expected Loss
1	109.82	43.927	2.00%	98.000%	0.879	0.9756	0.86
2	107.44	42.976	1.96%	96.040%	0.842	0.9518	0.80
3	105.00	42.000	1.92%	94.119%	0.807	0.9286	0.75

Based on the trader's analysis the bond is *most likely*:
A. correctly priced.
B. overvalued.
C. undervalued.

MODULE 28.3: CREDIT SCORES AND CREDIT RATINGS

Video covering this content is available online.

LOS 28.b: Explain credit scores and credit ratings.

Credit scoring is used for small businesses and individuals. A higher **credit score** indicates better credit quality. FICO is a well-known example of a credit scoring model used in the United States. FICO scores are higher for those with (a) longer credit histories (age of oldest account), (b) absence of delinquencies, (c) lower utilization (outstanding balance divided by available credit line), (d) fewer credit inquires, and (e) a wider variety of types of credit used.

Credit ratings are issued for corporate debt, asset-backed securities, and government and quasi-government debt. Similar to credit scores, credit ratings are ordinal ratings (higher = better). Three major global credit rating agencies are Moody's Investor Service, Standard & Poor's, and Fitch Ratings. The issuer rating for a company is typically for its senior unsecured debt. Ratings on other classes of debt by the same issuer may be notched. Notching is the practice of lowering the rating by one or more levels for more subordinate debt of the issuer. Notching accounts for LGD differences between different classes of debt by the same issuer (higher LGD for issues with lower seniority).

In addition to a letter grade, rating agencies provide an outlook (positive, negative, or stable). Higher-rated bonds trade at lower spreads relative to their benchmark

rates.

LOS 28.c: Calculate the expected return on a bond given transition in its credit rating.

Bond portfolio managers often want to evaluate the performance of a bond in the event of **credit migration** (i.e., change in rating). A change in a credit rating generally reflects a change in the bond's credit risk. The change in the price of the bond depends on the modified duration of the bond and the change in spread resulting from the change in credit risk as reflected by the credit migration.

$\Delta\%P = -$ (modified duration of the bond) × (Δ spread).

EXAMPLE: Credit migration

Suppose a bond with a modified duration of 6.32 gets downgraded from AAA to AA. The typical AAA credit spread is 60 bps, while the typical AA credit spread is 87 bps. Calculate the percentage change in price of the bond assuming that the bond is priced at typical spreads.

Answer:

change in spread = (87 – 60) = 27 bps = 0.0027.

$\Delta\%P = -$ (modified duration of the bond) × (Δ spread) = –6.32 × 0.0027 = –0.0171 or –1.71%.

MODULE QUIZ 28.3

1. Credit ratings incorporate:
   A. default probabilities only.
   B. loss given default only.
   C. both the default probability and the loss given default.
2. Credit scores and credit ratings are:
   A. ordinal.
   B. cardinal.
   C. optimized.
3. In credit rating, the practice of notching accounts for differences in:
   A. probability of default.
   B. business cycle impact.
   C. loss given default.
4. Bjørn Johansen, portfolio manager for Agnes Advisors, is concerned about one of the holdings in Agnes's fixed income portfolio. Beta, Inc., bonds are currently rated A, but with a negative outlook. Beta bonds have a modified duration of 9.20. Johansen collects the following information about average spreads by ratings class.

Rating	AAA	AA	A	BBB	BB	B	CCC/ CC/C
Avg. Spread	0.24%	0.32%	0.49%	0.60%	0.77%	0.95%	1.22%

   Assuming that the spread on Beta bonds is equal to the average spread in its rating class, what is the expected change in price of Beta, Inc., bonds if Beta gets downgraded to BBB?
   A. –1.01%.

B. –0.82%.
C. –0.92%.

MODULE 28.4: STRUCTURAL AND REDUCED FORM MODELS

Video covering this content is available online.

LOS 28.d: Explain structural and reduced-form models of corporate credit risk, including assumptions, strengths, and weaknesses.

STRUCTURAL MODELS

Structural models of corporate credit risk are based on the structure of a company's balance sheet and rely on insights provided by option pricing theory.

Option Analogy

Consider a hypothetical company with assets that are financed by equity and a single issue of zero-coupon debt. The value of the assets at any point in time is the sum of the value of equity and the value of debt.

Due to the limited liability nature of corporate equity, the shareholders effectively have a call option on the company's assets with a strike price equal to the face value of debt. If at the maturity of the debt, the value of the company's assets is higher than the face value of debt, shareholders will exercise their call option to acquire the assets (and then pay off the debt and keep the residual). On the other hand, if the value of the company's assets is less than the face value of debt, the shareholders will let the option expire worthless (i.e., default on the debt), leaving the company's assets to the debt investors.

Hence, at time *T*, (corresponding to the maturity of debt):

$$\text{value of equity}_T = \max(0, A_T - K)$$

$$\text{value of debt} = A_T - \text{value of equity} = A_T - \max(0, A_T - K)$$

which means that value of $\text{debt}_T = \min(A_T, K)$

where:

A_T = value of company's assets at time *T* (i.e., at maturity of debt)

K = face value of debt

An alternate, but related interpretation considers equity investors as long the net assets of the company (with a time *T* value of $A_T - K$) and long a put option, allowing them to sell the assets at an exercise price of K. Default is then synonymous to exercising the put option.

$$\text{value of the put option} = \max(0, K - A_T)$$

Under the put option analogy, the investors in risky debt can be construed to have a long position in risk-free debt and a short position in that put option.

$$\text{value of risky debt} = \text{value of risk-free debt} - \text{value of put option}$$

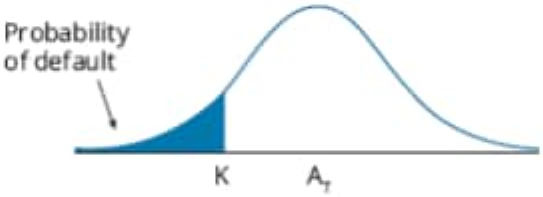
From our earlier discussion on CVA, recall that:

value of risky debt = value of risk-free debt – CVA

Therefore, the value of the put option = CVA.

Figure 28.2 shows the distribution of asset values at time *T*. If the value of assets falls below the default barrier K, the company defaults. The probability of default is indicated by the region in the left tail below the default barrier of K.

Figure 28.2: Distribution of Asset Value at Time T



Advantages of structural models:

1. Structural models provide an economic rationale for default (i.e., $A_T < K$) and explain why default occurs.
2. Structural models utilize option pricing models to value risky debt.

Disadvantages of structural models:

1. Because structural models assume a simple balance sheet structure, complex balance sheets cannot be modeled. Additionally, when companies have off-balance sheet debt, the default barrier under structural models (K) would be inaccurate and hence the estimated outputs of the model will be inaccurate.
2. One of the key assumptions of the structural model is that the assets of the company are traded in the market. This restrictive assumption makes the structural model impractical.

REDUCED FORM MODELS

Reduced form (RF) models do not rely on the structure of a company's balance sheet and therefore do not assume that the assets of the company trade. Unlike the structural model, reduced form models do not explain *why* default occurs. Instead, they statistically model *when* default occurs. Default under the RF model is a randomly occurring *exogenous* variable. As such, the RF model imposes assumptions on the output of the structural model (asset values, recovery rates, etc.).

A key input into the RF model is the **default intensity**, which is the probability of default over the next (small) time period. Default intensity can be estimated using regression models. These regression models employ several independent variables including company specific variables (e.g., leverage, beta, interest coverage ratio) as well as macro-economic variables.

Advantages of reduced form models:

1. RF models do not assume that the assets of a company trade.
2. Default intensity is allowed to vary as company fundamentals change, as well as when the state of the economy changes.

Disadvantages of reduced form models:

1. RF models do not explain why default occurs.
2. Under RF models, default is treated as a random event (i.e., a surprise), but in reality, default is rarely a surprise (it is often preceded by several downgrades).

MODULE QUIZ 28.4

1. The probability of default under the structural model is *most likely*:
   A. endogenous.
   B. exogenous.
   C. estimated using a regression model.
2. Under the structural model, risky debt can be thought of as equivalent to a portfolio comprising a long position in a risk-free bond and:
   A. a short put option on assets of the company, with a strike price equal to the face value of debt.
   B. a long put option on the assets of the company, with a strike price equal to the face value of debt.
   C. short put option on assets of the company, with a strike price equal to the present value of debt.
3. Which one of the following is *least likely* a disadvantage of structural models?
   A. Structural models are not appropriate in the presence of off-balance sheet financing.
   B. Structural models assume that company assets trade.
   C. Structural models do not explain why default occurs.
4. Default intensity is *least likely* to be:
   A. estimated using company specific and macroeconomic variables.
   B. the probability of default over the next time period.
   C. constant over the life of the risky bond.

MODULE 28.5: CREDIT SPREAD ANALYSIS

Video covering this content is available online.

LOS 28.e: Calculate the value of a bond and its credit spread, given assumptions about the credit risk parameters.

credit spread on a risky bond = YTM of risky bond – YTM of benchmark

The value of a risky bond, assuming it does not default, is its **value given no default (VND)**. VND is calculated using the risk-free rate to value the risky bond.

EXAMPLE: Credit spread

Jack Gordon, a fixed income analyst for Omega Bank PLC, is evaluating an AA corporate bond for inclusion in the bank's portfolio. The $100 par, 3.50%, annual-pay, 5-year bond is currently priced with a credit spread of 135 bps over the benchmark par rate of 2%. Calculate the bond's CVA implied in its market price.

Answer:

VND for the bond = present value of bond's cash flows using benchmark YTM.

N = 5, PMT = 3.50, I/Y = 2, FV = 100, CPT PV = 107.07

value of the risky bond using credit spread = 2% benchmark rate + 1.35% credit spread = 3.35% as YTM:

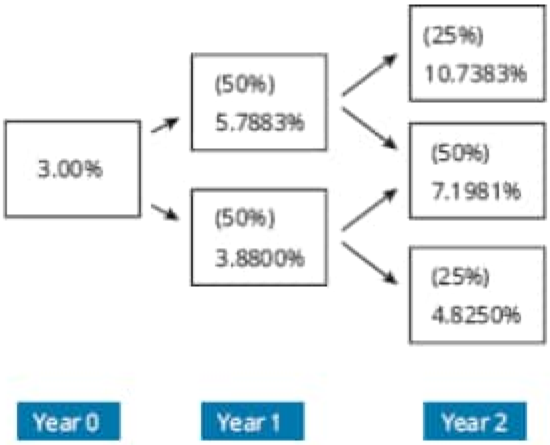
FV = 100, N = 5, PMT = 3.5, I/Y = 3.35, CPT PV = 100.68

CVA = VND – value of risky debt = 107.07 – 100.68 = $6.39

In our earlier discussion on the computation of CVA, we assumed zero volatility for benchmark rates. We now introduce volatility in future 1-period benchmark rates using the binomial tree framework.

Figure 28.3 shows a three-year benchmark interest rate tree. Numbers in parentheses in each node indicate the probability for that node (note that probabilities add up to 100% for each period).

Figure 28.3: Benchmark Interest Rate Tree



We use our standard backward induction process to value a risky bond given the benchmark rate tree. Recall that for a binomial interest rate tree, the probability of up = probability of down = 0.5. Because the rates in the tree are benchmark rates, the value obtained is the bond's VND.

We illustrate the computation of VND using the following example.

EXAMPLE: Calculation of VND

For a 3-year, annual pay, 4% coupon, $100 par corporate bond using the interest rate tree in Figure 28.3, calculate the VND for the bond and the expected exposure for each year.

Answer:

The completed tree is shown as follows:

Given a 4% coupon, the cash flow at the end of three years is 104. The value at the bottom node in Year 2 = 104 / 1.04825 = $99.21. The value of bottom node in Year 1 is calculated as the present value of the average of the two values in Year 2 plus the coupon.

([(99.21 + 97.02) / 2] + 4) / (1.0388) = 98.30.

Using the same procedure for Year 0, the VND for the bond is $97.24.

expected exposure for Year t = Σ (value in node i at time t × node probability) + coupon for Year t

expected exposure for Year 1 = (0.5)(98.30) + (0.5)(94.02) + 4 = $100.16

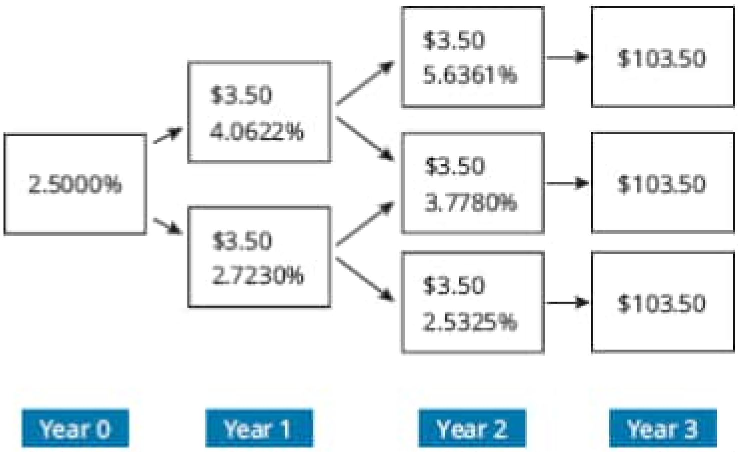
expected exposure for Year 2 = (0.25)(93.92) + (0.5)(97.02) + (0.25)(99.21) + 4 = $100.79

expected exposure for Year 3 = $104 (no need to calculate!)

Once the expected exposure for each period is calculated, given an estimated unconditional probability of default and a recovery rate, we use the same method discussed earlier to calculate the CVA for the bond.

MODULE QUIZ 28.5

1. An analyst has completed the following interest rate tree:



The VND for a 3-year, annual-pay, 3.50% corporate bond is *closest* to:

A. $99.78.
B. $100.70.
C. $101.54.

2. Jill Smith, CFA, is evaluating an annual pay 4%, 3-year corporate bond and has compiled the following information. Smith has assumed a recovery rate of 60% and a flat benchmark rate of 2.50%. Unfortunately, some of the information is missing. Complete the missing information to answer the question.

Year	Exposure	LGD	PD	PS	Expected Loss	DF	PV of Expected Loss
1	106.89	42.756	1.00%	99.000%	0.428	0.9756	0.42
2			0.99%	98.010%		0.9518	
3	104.00	41.600	0.98%	97.030%	0.408	0.9286	0.38

The CVA on the bond is *closest* to:

A. $1.20.

B. $2.58.

C. $2.82.

3. Jill Smith, CFA, is evaluating a 4%, annual-pay, 3-year corporate bond with a CVA of $1.88. The bond is trading at $103.53. The benchmark par curve is given here.

Year	1	2	3	4	5
Par Rate	2%	2.25%	2.50%	2.75%	3.00%

Smith is *most likely* to conclude that the corporate bond is:

A. overvalued.

B. undervalued.

C. correctly valued.

MODULE 28.6: CREDIT SPREAD

Video covering this content is available online.

LOS 28.f: Interpret changes in a credit spread.

A benchmark yield should be equal to the real risk-free rate plus expected inflation, as well as a risk-premium for uncertainty in future inflation. **Credit spreads** include compensation for default, liquidity, and taxation risks relative to the benchmark. Adjustment to the price for all these risk factors together is known as the *XVA*. In this reading, we focus only on the default risk component (i.e., CVA) as it is the most important and most-commonly-used in practice.

Credit spreads change as investor perceptions about the future probability of default and recovery rates change. These perceptions depend on expectations about the state of the economy. Expectations of impending recessions lead to expectations of higher defaults and lower recovery rates.

EXAMPLE: Changes in probability of default and recovery rates

Joan De Silva, a junior analyst, works for a regional bank. Currently, De Silva is evaluating a 3-year, annual pay, 3% XYZ corporate bond priced at $102. The benchmark yield curve is flat at 1.75%.

De Silva prepares Table 1 assuming a hazard rate of 1.25% and a recovery rate of 70%.

Table 1: Credit Analysis of 3%, 3-Year Bond With 1.25% PD and 70% RR

Year	Exposure	LGD	PD	PS	Expected Loss	DF	PV of Expected Loss
1	105.44	31.63	1.25%	98.750%	0.395	0.983	0.389
2	104.23	31.27	1.23%	97.516%	0.386	0.966	0.373
3	103.00	30.90	1.22%	96.297%	0.377	0.949	0.358
						CVA	1.12

Expected exposure is the amount of money a bond investor in a credit risky bond stands to lose at a point in time before any recovery is factored in.

The expected exposure for each period is calculated given the coupon of $3 and risk-free rate of 1.75%:

expected exposure for Year 1 = (104.23 / 1.0175) + 3 = $105.44

expected exposure for Year 2 = (103 / 1.0175) + 3 = $104.23

expected exposure for Year 3 = $103 (no need to calculate)

De Silva prepares a report including Table 1 for Susan Collins, Chief Investment Officer. Collins states that based on the expectations of a slowdown in the economy, a 1.50% hazard rate and a recovery rate of 60% would be more appropriate for XYZ.

De Silva updates those estimates and prepares Table 2.

Table 2: Credit Analysis of 3%, 3-Year Bond With 1.50% PD and 60% RR

Year	Exposure	LGD	PD	PS	Expected Loss	DF	PV of Expected Loss
1	105.44	42.174	1.50%	98.500%	0.633	0.983	0.622
2	104.23	41.691	1.48%	97.023%	0.616	0.966	0.595
3	103.00	41.20	1.46%	95.567%	0.600	0.949	0.569
						CVA	1.79

1. Using De Silva's estimates of hazard rate and recovery rate, XYZ bond is currently *most likely*:
   A. undervalued.
   B. overvalued.
   C. fairly priced.

2. Using the market price of the bond, the credit spread on XYZ bond is *closest* to:
   A. 0.44%.
   B. 0.49%.
   C. 0.55%.

3. Assuming the market price changes to reflect Collins's expectations of PD and recovery rate, the new credit spread would be *closest* to:
   A. 0.52%.
   B. 0.61%.
   C. 0.79%.

Answer:

1. **A** Using the benchmark rate, XYZ bond's VND is calculated as:

    N = 3, PMT = 3, I/Y = 1.75, FV = 100, CPT PV = 103.62

    value of risky bond = VND – CVA = 103.62 – 1.12 = 102.50

    The market price of $102 for the bond implies that the bond is undervalued.

2. **C** YTM for 3-year risk-free bond = 1.75% (given)

    YTM for XYZ bond: PV = –102, N = 3, PMT = 3, FV = 100, CPT I/Y = 2.30%

    credit spread = 2.30 – 1.75 = 0.55%

3. **B** Based on the revised PD and recovery rate, CVA = 1.79.

    value of risky bond = VND – CVA = 103.62 – 1.79 = 101.83

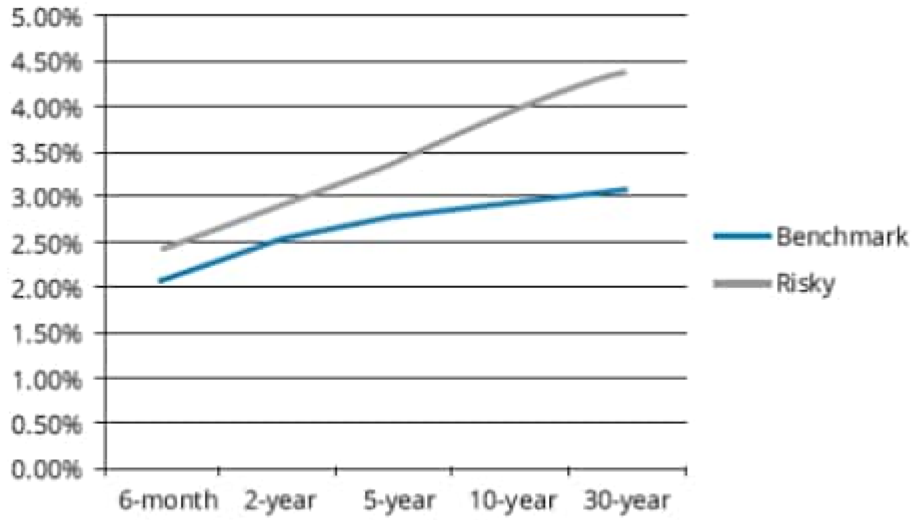
    YTM for XYZ bond: PV = –101.83, N = 3, PMT = 3, FV = 100, CPT I/Y = 2.36%

    credit spread = 2.36 – 1.75 = 0.61%

LOS 28.g: Explain the determinants of the term structure of credit spreads and interpret a term structure of credit spreads.

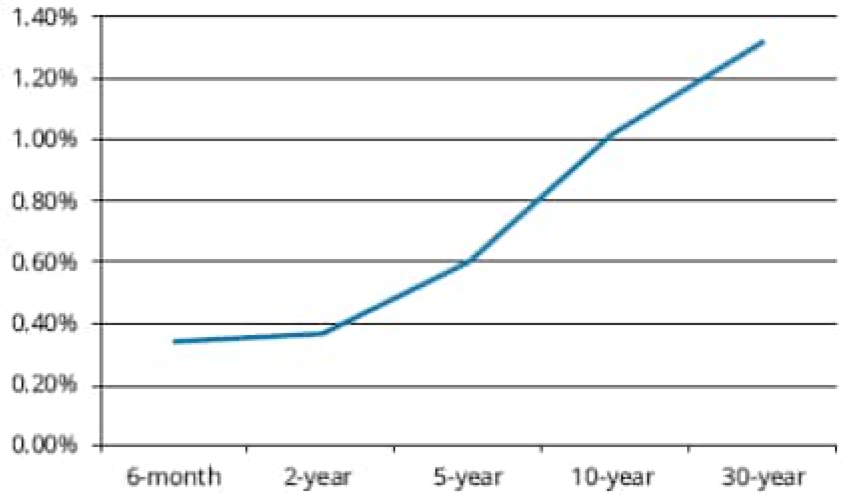
The **term structure of credit spreads** shows the relationship between credit spreads and maturity. Term structures can be for different bonds issued by the same issuer (with varying maturities) or different bonds within a sector (e.g., AA term structure). Term structures can be useful for investors to price a new issue, as well as to determine the relative valuation of an existing issue. A **credit spread curve** displays this relationship graphically. Figure 28.4 shows yields on U.S. Treasury bonds and high-quality (AAA) corporate bonds. Figure 28.5 shows the resulting HQM spread curve.

Figure 28.4: U.S. Treasury and High-Quality Corporate Bond Yields



Source: St. Louis Federal Reserve Bank, May 2018.

Figure 28.5: Credit Spread Curve: High-Quality Corporates



The credit spread is inversely related to the recovery rate and positively related to the probability of default. Recall that credit spread is calculated as the difference between the yields of a risky security and a benchmark security with the same maturity. However, oftentimes a maturity-matched, liquid (i.e., frequently traded) benchmark security is not available. In such cases, interpolation from available benchmark securities is used. When such data is not easily available, or when benchmark bonds are thinly traded, a spread relative to a swap fixed rate (corresponding to the maturity of the bond) may be used.

To create a spread curve, all of the bonds whose spreads are used should have similar credit characteristics. Differences in seniority, first/second lien provisions, and embedded options distort the computed credit spread. A spread curve using these distorted spreads would not be an accurate reflection of the term structure.

Key determinants of the shape of the credit spread curve include expectations about future recovery rates and default probabilities. If default probabilities are expected to be higher (or recovery rates lower) in the future, the credit curve would be expected to be positively sloped. Flat credit curves indicate stable expectations over time.

Determinants of Term Structure of Credit Spreads

1. *Credit quality* is an important factor driving term structure; AAA term structures tend to be flat or slightly upward sloping. Lower-rated sectors tend to have steeper spread curves, reflecting greater uncertainty as well as greater sensitivity to the business cycle.
2. *Financial conditions* affect the credit spread curve. Spreads narrow during economic expansions and widen during cyclical downturns. During boom times, benchmark yields tend to be higher while credit spreads tend to be narrower.
3. *Market demand and supply* influence the shape of the spread curve. Recall that a credit spread includes a premium for lack of liquidity. Hence, less liquid maturities would show higher spreads (even if the expectations for that time period are stable). Due to low liquidity in most corporate issues, the credit curves are most heavily influenced by more heavily traded bonds. Because newly issued bonds are generally more liquid, when an issuer refinances a near-dated bond with a longer-term bond, the spread may appear to narrow for the longer maturity (possibly leading to an inverted credit spread curve).

4. *Equity market volatility*: Company-value models (structural models discussed earlier) employ a company's stock price volatility and balance sheet structure in determining the probability of default. Increases in equity volatility therefore tend to widen spreads and influence the shape of the credit spread curve.

MODULE QUIZ 28.6

1. Mossimo Gulzar, is compiling data on spreads for AAA corporate bonds. Mossimo has compiled the following information from the firm's senior economist.

Year	1	3	5	10
Benchmark Par Rate	0.75%	1.25%	1.75%	2.25%
AAA CVA	$1.79	$2.12	$3.24	$5.84

Using a typical 4% coupon AAA bond for each maturity category, Mossimo would *most likely* conclude that the credit spread curve is:
   A. upward sloping.
   B. downward sloping.
   C. flat.

2. Credit spreads are *most likely*:
   A. positively related to probability of default and loss severity.
   B. positively related to probability of default and recovery rate.
   C. negatively related to probability of default and recovery rate.

3. Credit spread curves of the highest rated bond sectors tend to be:
   A. flat or downward sloping.
   B. flat or slightly upward sloping.
   C. steeply upward sloping.

4. Flat credit curves are *most likely* indicative of:
   A. expectations of economic expansion.
   B. expectations of a recession.
   C. stable expectations about future recovery rates and default probabilities.

5. The shape of the credit curve is *least likely* to be affected by:
   A. demand and supply forces in the market.
   B. sector quality.
   C. the swap rate curve.

MODULE 28.7: CREDIT ANALYSIS OF SECURITIZED DEBT

Video covering this content is available online.

LOS 28.h: Compare the credit analysis required for securitized debt to the credit analysis of corporate debt.

Securitized debt entails financing of specific assets (e.g., auto loans, credit card receivables, and mortgages) without financing the entire balance sheet (which would be the case with the general obligation nature of corporate and sovereign bonds). Secured debt is usually financed via a bankruptcy-remote SPE. This isolation of securitized assets allows for higher leverage and lower cost to the issuer. Investors also benefit from greater diversification, more stable cash flows and a higher risk premium relative to similarly-rated general obligation bonds (due to higher complexity associated with collateralized debt).

Components of Credit Analysis of Secured Debt

1. **Collateral pool:** Credit analysis of structured, securitized debt begins with the collateral pool. Homogeneity of a pool refers to the similarity of the assets within the collateral pool. Granularity refers to the transparency of assets within the pool. A highly granular pool would have hundreds of clearly defined loans, allowing for use of summary statistics as opposed to investigating each borrower. A more-discrete pool of a few loans would warrant examination of each obligation separately.

   Short-term granular and homogenous structured finance vehicles are evaluated using a statistical-based approach. Medium-term granular and homogenous obligations are evaluated using a portfolio-based approach because the portfolio composition varies over time. Discrete and non-granular portfolios have to be evaluated at the individual loan level.

2. **Servicer quality** is important to evaluate the ability of the servicer to manage the origination and servicing of the collateral pool. After origination, investors in secured debt face the operational and counterparty risk of the servicer. A servicer's past history is often used as an indication of servicer quality.

3. **Structure** determines the tranching or other management of credit and other risks in a collateral pool. One key structural element is credit enhancement, which may be internal or external.

   Examples of internal credit enhancements include tranching of credit risk among classes with differing seniority (i.e., the distribution waterfall), overcollateralization, and excess servicing spread (whereby such excess collateral or spread becomes the first line of defense against credit losses). Third party guarantees (e.g., bank, insurance companies, or loan originators) are an example of external credit enhancements.

   A special structure is the case of a **covered bond**. Issued by a financial institution, covered bonds are senior, secured bonds backed by a collateral pool *as well as* by the issuer (i.e., covered bond investors have recourse rights). Originated in Germany, covered bonds have spread to the rest of Europe, Asia, and Australia. While the collateral (known as the cover pool) types vary by jurisdiction, common forms are commercial and residential mortgages and public sector loans. The cover pool is dynamic; the sponsor has to replace any prepaid or nonperforming asset in the pool so that the pool generates sufficient cash flow to service the bonds.

   In the event the financial institution sponsor misses a payment, a soft or a hard bullet may be triggered. In the case of soft-bullet covered bonds, the maturity date of the bond is extended up to one year and all payments are not immediately due. In the hard-bullet covered bonds, the payments are accelerated.

MODULE QUIZ 28.7

1. Short-term granular and homogenous structured finance vehicles are *most appropriately* evaluated using a:
   A. statistical-based approach.
   B. portfolio-based approach.
   C. loan level analysis.

2. Medium-term granular and homogenous structured finance vehicles are *most appropriately* evaluated using a:
   A. statistical-based approach.
   B. portfolio-based approach.
   C. loan level analysis.
3. With respect to the servicer in a secured debt, investors *most likely* face:
   A. operational and financial risk only.
   B. financial and counterparty risk only.
   C. operational and counterparty risk only.
4. Covered bonds are *most likely* issued by a financial institution and:
   A. are backed by the government.
   B. are backed by the collateral pool only.
   C. have recourse rights as well as backing of the collateral pool.

KEY CONCEPTS

LOS 28.a

Expected exposure is the amount of money a bond investor in a credit risky bond stands to lose at a point in time *before* any recovery is factored in. Loss given default is equal to loss severity multiplied by exposure. Credit valuation adjustment (CVA) is the sum of the present values of expected losses for each period.

LOS 28.b

Credit scoring is used for small businesses and individuals. Credit ratings are issued for corporate debt, asset-backed securities, and government and quasi-government debt. Credit scores and ratings are ordinal rankings (higher = better). Notching different issues of the same issuer based on seniority ensures that credit ratings incorporate the probability of default as well as loss given default.

LOS 28.c

The change in the price of a bond resulting from credit migration depends on the modified duration of the bond and the change in spread.

$$\Delta\%P = -(\text{modified duration of the bond}) \times (\Delta \text{ spread})$$

LOS 28.d

Structural models of corporate credit risk are based on the structure of a company's balance sheet and rely on insights provided by option pricing theory. Structural models consider equity as a call option on company assets. Alternately, an investment in a risky bond can be viewed as equivalent to purchasing a risk-free bond and writing a put option on company assets.

Unlike the structural model, reduced-form models do not explain why default occurs. Instead, they statistically model when default occurs. Default under a reduced-form model is a randomly occurring exogenous variable.

LOS 28.e

credit spread on a risky bond = YTM of risky bond – YTM of benchmark

The value of a risky bond, assuming it does not default, is its value given no default (VND). VND is calculated using the risk-free rate to value the risky bond.

A backward induction procedure using a risky bond's cash flows and a benchmark interest-rate tree can also be used to calculate the bond's VND.

LOS 28.f

Credit spreads change as investors' perceptions change about the future probability of default and recovery rates. Expectations of impending recessions lead to expectations of higher default and lower recovery rates.

LOS 28.g

The term structure of credit spreads shows the relationship between credit spreads and maturity. Term structure depends on credit quality, financial conditions, demand and supply in the bond market, and expected volatility in the equity markets.

LOS 28.h

Credit analysis of ABS involves the analysis of the collateral pool, servicer quality, and the structure of secured debt (i.e., distribution waterfall).

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 28.1

1. **A** Value of benchmark 5% annual pay, 5-year bond (using benchmark rate of 2%): N = 5, PMT = 5, FV = 100, I/Y = 2, CPT PV = 114.14

   Value of Barry Corp. bonds would be lower by its CVA = 114.14 – 12.67 = \$101.47.

   (LOS 28.a)

2. **B** The hazard rate (the conditional probability of default) will be equal to the probability of default in the first year. The probability of default will be less than the hazard rate in all years after the first. (LOS 28.a)

3. **C** Probability of survival is the probability that the bond does not default. For Year 3, the probability of survival is PS = $(1 - 0.0125)^3$ = 96.30%. (LOS 28.a)

4. **B** expected loss = exposure × (1 – recovery rate) × PD

   exposure × (1 – recovery rate) = loss given default (LGD)

   Hence, expected loss = LGD × PD. (LOS 28.a)

5. **A** CVA = price of risk-free bond – price of risky bond

   CVA is the sum (not the mean) of the present value of expected loss. (LOS 28.a)

6. **A** Risk-neutral probability of default is the probability of default that is priced by the market. If Abramson's estimated hazard rate is less than this, the market has priced-in a higher credit risk in the bond (i.e., the bond is undervalued). (LOS 28.a)

7. **A** Given the market price, if the assumed recovery rate is increased, risk-neutral probability of default (that is implied in the current market price) has to be higher to offset the higher recovery rate. As such, the assumed recovery rate is positively correlated with the risk-neutral probability of default. (LOS 28.a)

Module Quiz 28.2

1. **C** exposure in Year 2 = 100 / 1.03 = 97.09

   recovery cash flow = exposure × recovery rate = 97.09 × 0.45 = 43.69

   PV = –89.49, N = 2, FV = 43.69, CPT I/Y = –30.13%

   (LOS 28.a)

2. **B** Bond Q has the lowest expected loss and hence the lowest credit risk.

Bond	Exposure (per $100 par)	Recovery (per $100 par)	Probability of Default	LGD (per $100 par)	Expected Loss
P	98	50	2.50%	48	1.20
Q	94	56	3.00%	38	1.14
R	89	49	4.65%	40	1.86

   (LOS 28.a)

3. **C** The value of a comparable risk-free bond (using the benchmark rate of 2.50%) can be calculated to be $107.41 as follows:

   N = 3, PMT = 5, FV = 100, I/Y = 2.50, CPT PV = 107.14.

   The sum of the PV of expected loss column is the CVA = 2.41.

   Hence, the risky bond value = 107.41 – 2.41 = $104.73, which is greater than the market price of $103. The bond is undervalued. (LOS 28.a)

Module Quiz 28.3

1. **C** Due to the practice of notching (lower ratings for subordinated debt as compared to senior debt of the same issuer), the loss given default is incorporated into credit ratings along with the probability of default. (LOS 28.b)

2. **A** Credit scores and credit ratings are ordinal measures; a higher rating implies better credit risk, but the difference in scores or ratings is not proportional to the difference in risk. (LOS 28.b)

3. **C** Notching accounts for LGD differences between different classes of debt by the same issuer (higher LGD for issues with lower seniority). (LOS 28.b)

4. **A** The change in spread from A to BBB is 0.60% – 0.49% = +0.11%. The percent change in price = –9.20 × 0.11% = –1.01%. (LOS 28.c)

Module Quiz 28.4

1. **A** Probability of default under the structural model is endogenous and hence is not estimated using a regression model. It is the probability that the uncertain future value of the company assets is below the default barrier. (LOS 28.d)

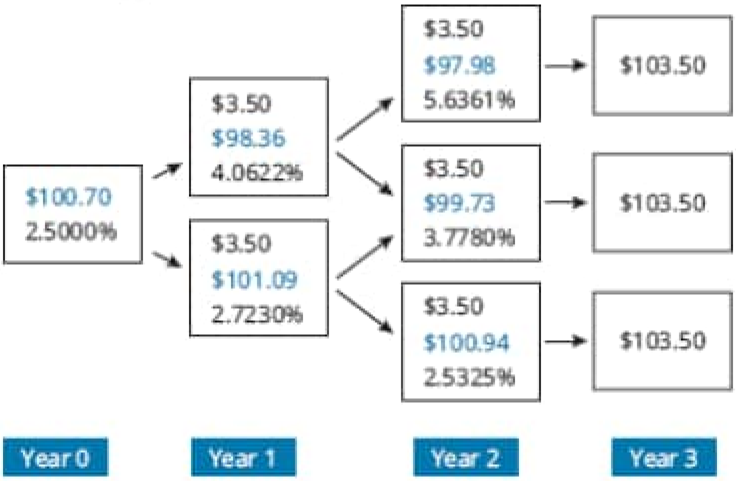
2. **A** Because of limited liability of equity investors, risky debt investors are effectively short a put option on company assets with a strike price equal to the face value of debt. Hence, a risky debt investment is equivalent to an investment in risk-free debt as well as a short put option on company assets. (LOS 28.d)

3. **C** Disadvantages of structural models include (1) SMs assume that assets of the company trade and (2) SMs are inappropriate when the balance sheet is complex or there are off-balance sheet liabilities. Structural models explain why default occurs; reduced form models do not. (LOS 28.d)

4. **C** Default intensity is the probability of default over the next time increment. Default intensity varies over the life of the risky bond as company circumstances and the state of the economy change. (LOS 28.d)

Module Quiz 28.5

1. **B** The completed tree is shown here.



VND = [(101.09 + 98.36) / 2 + 3.50] / 1.025 = $100.70

(LOS 28.e)

2. **A** We need to calculate the expected exposure, LGD, expected loss, and PV of expected loss for Year 2. The coupon of $4 and risk-free rate of 2.5% give expected exposure in Year 2 = (104 / 1.025) + 4 = $105.46.

 LGD in Year 2 = exposure × (1 – recovery rate) = 105.46 × 0.4 = 42.19

 expected loss = LGD × PD = 42.19 × 0.0099 = $0.418

 PV of expected loss = expected loss × DF = 0.42 × 0.9518 = $0.40

 CVA = sum of PV of expected loss = 0.42 + 0.40 + 0.38 = $1.20

 (LOS 28.e)

3. **A** Using the 3-year par rate, we can calculate the VND for the corporate bond as:

 FV = 100, N = 3, PMT = 4, I/Y = 2.50, CPT PV = 104.28

 value of the corporate bond = VND – CVA = 104.28 – 1.88 = $102.40

 Since the bond is trading at $103.53, it is overvalued. (LOS 28.e)

Module Quiz 28.6

1. **B** The coupon rate is given as 4%. The following table shows the results:

Year	1	3	5	10
Par Rate	0.75%	1.25%	1.75%	2.25%
AAA CVA	$1.79	$2.12	$3.24	$5.84
VND	$103.23	$108.05	$110.68	$115.52
V (Risky)	$101.44	$105.93	$107.44	$109.68
Yield (Risky)	2.53%	1.95%	2.40%	2.88%
Credit Spread	1.78%	0.70%	0.65%	0.63%

VND is calculated using the benchmark par rate. For example, VND for 5-year bond: N = 5, FV = 100, PMT = 4, I/Y = 1.75, CPT PV = 110.68

value (risky) = VND – CVA

Yield (risky) is computed using the value of risky bond as its price.

Yield (risky) for 3-year bond: N = 3, PV = –105.93, PMT = 4, FV = 100, CPT I/Y = 1.95%

credit spread = yield (risky) – par rate

Credit spreads appear to decline with maturity, resulting in a downward sloping credit spread curve. (LOS 28.f)

2. **A** Credit spreads are positively related to probability of default and loss severity. Loss severity = 1 – recovery rate. (LOS 28.g)

3. **B** The highest rated bond sectors tend to have a flat or slightly upward sloping credit spread curve. (LOS 28.g)

4. **C** When expectations about probabilities of default and recovery rates are stable, credit spread curves tend to be flat. (LOS 28.g)

5. **C** The shape of the credit spread curve is determined by sector quality, market demand and supply forces, company-value models, and financial conditions in the economy. (LOS 28.g)

Module Quiz 28.7

1. **A** Short-term granular and homogenous structured finance vehicles are evaluated using a statistical-based approach. (LOS 28.h)

2. **B** Medium-term granular and homogenous obligations are evaluated using a portfolio-based approach because the portfolio composition varies over time. (LOS 28.h)

3. **C** After origination, investors in a secured debt face operational and counterparty risk of the servicer. (LOS 28.h)

4. **C** Covered bonds are senior, secured bonds issued by a financial institution. Covered bonds are backed by a collateral pool *as well as* by the issuer (i.e., covered bond investors have recourse rights). (LOS 28.h)

READING 29

CREDIT DEFAULT SWAPS

EXAM FOCUS

A credit default swap (CDS) is a contract between two parties in which one party purchases protection from another party against losses from the default of a borrower. For the exam, you should be able to describe CDS, as well as related securities like index CDS. You should know what a credit event is and how the different protocols for settlement work. You should be familiar with the principles and factors that drive market pricing of CDS. Be able to describe how CDS are used to manage credit exposure, and how they can be used to profit from anticipated changes in the credit curve. You should understand how CDS are used for arbitrage to take advantage of relative mispricings of different risky securities.

MODULE 29.1: CDS FEATURES AND TERMS

Video covering this content is available online.

CREDIT DEFAULT SWAPS

A **credit default swap** *(CDS)* is essentially an insurance contract. If a credit event occurs, the *credit protection buyer* gets compensated by the *credit protection seller*. To obtain this coverage, the protection buyer pays the seller a premium called the **CDS spread**. The protection seller is assuming (i.e., long) credit risk, while the protection buyer is short credit risk. Note that the CDS does not provide protection against market-wide interest rate risk, only against credit risk. The contract is written on a face value of protection called the **notional principal** (or "notional").

PROFESSOR'S NOTE

The terminology related to CDS is counterintuitive: the protection **buyer** is the short party (short the credit risk of the reference asset and short the CDS), while the **seller** is the long party (long the CDS and long the credit risk of the reference asset).

Even though the CDS spread should be based on the underlying credit risk of the reference obligation, standardization in the market has led to a fixed **coupon** on CDS products: 1% for investment-grade securities and 5% for high-yield securities. Hence, the coupon rate on the CDS and the actual credit spread may be different. The present value of the difference between the standardized coupon rate and the credit spread on the reference obligation is paid upfront by one of the parties to the contract. For example, a CDS on an investment-grade bond with a credit spread of 75 basis points (bps) would require a premium payment of 100bps (CDS coupon rate) by the protection buyer. To compensate the protection buyer (who pays a higher-

than-market premium), the protection seller would then pay upfront to the buyer the present value of 25bps of the notional principal.

For a protection buyer, a CDS has some of the characteristics of a put option—when the underlying performs poorly, the holder of the put option has a right to exercise the option.

The **International Swaps and Derivatives Association (ISDA)**, the unofficial governing body of the industry, publishes standardized contract terms and conventions in the **ISDA Master Agreement** to facilitate smooth functioning of the CDS market.

LOS 29.a: Describe credit default swaps (CDS), single-name and index CDS, and the parameters that define a given CDS product.

SINGLE-NAME CDS

In the case of a single-name CDS, the **reference obligation** is the fixed-income security on which the swap is written, usually a senior unsecured obligation (in the case of a **senior CDS**). The issuer of the reference obligation is called the **reference entity**. The CDS pays off not only when the reference entity defaults on the reference obligation but also when the reference entity defaults on any other issue that is ranked pari passu (i.e., same rank) or higher. The CDS payoff is based on the market value of the **cheapest-to-deliver (CTD)** bond that has the same seniority as the reference obligation.

PROFESSOR'S NOTE

The cheapest-to-deliver bond is the debt instrument with the same seniority as the reference obligation but that can be purchased and delivered at the lowest cost.

EXAMPLE: Cheapest-to-deliver

Party X is a protection buyer in a $10 million notional principal senior CDS of Alpha, Inc. There is a credit event (i.e., Alpha defaults) and the market prices of Alpha's bonds after the credit event are as follows:

- Bond P, a subordinated unsecured debenture, is trading at 15% of par.
- Bond Q, a five-year senior unsecured debenture, is trading at 25% of par.
- Bond R, a three-year senior unsecured debenture, is trading at 30% of par.

What will be the payoff on the CDS?

Answer:

The cheapest-to-deliver senior unsecured debenture (i.e., same seniority as the senior CDS) is bond Q. The payoff will be the difference between the notional principal and market value of the CTD.

payoff = $10 million − (0.25)($10 million) = $7.5 million.

INDEX CDS

An *index CDS* covers multiple issuers, allowing market participants to take on an exposure to the credit risk of several companies simultaneously in the same way that stock indexes allow investors to take on an equity exposure to several companies at once. In this case, the protection for each issuer is equal (i.e., equally weighted) and the total notional principal is the sum of the protection on all the issuers.

EXAMPLE: Index CDS

Party X is a protection buyer in a five-year, $100 million notional principal CDS for CDX-IG, which contains 125 entities. One of the index constituents, company A, defaults and its bonds trade at 30% of par after default.

1. What will be the payoff on the CDS?
2. What will be the notional principal of the CDS after default?

Answer:

1. The notional principal attributable to entity A is $100 million / 125 = $0.8 million. Party X should receive payment of $0.8 million – (0.3)($0.8 million) = $560,000.
2. Post the default event, the remainder of the CDS continues with a notional principal of $99.2 million.

The pricing of an index CDS is dependent on the correlation of default (credit correlation) among the entities in the index. The higher the correlation of default among index constituents, the higher the spread on the index CDS.

LOS 29.b: Describe credit events and settlement protocols with respect to CDS.

A default is defined as the occurrence of a credit event. Common types of credit events specified in CDS agreements include bankruptcy, failure to pay, and restructuring.

- *Bankruptcy.* A bankruptcy protection filing allows the defaulting party to work with creditors under the supervision of the court so as to avoid full liquidation.
- *Failure to pay.* Occurs when the issuer misses a scheduled coupon or principal payment without filing for formal bankruptcy.
- *Restructuring.* Occurs when the issuer forces its creditors to accept terms that are different than those specified in the original issue. Restructuring is less common in the United States as issuers prefer to go the bankruptcy protection route.

A 15-member group of the ISDA called the **Determinations Committee (DC)** declares when a credit event has occurred. A supermajority vote (at least 12 members) is required for a credit event to be declared.

When there is a credit event, the swap will be settled in cash or by physical delivery. With physical delivery, the protection seller receives the reference obligation (i.e.,

the bond or loan) and pays the protection buyer the notional amount, as shown in Figure 29.1.

Figure 29.1: Physical Settlement on Credit Default Swap After a Credit Event

In the case of a cash settlement, the payout amount is the payout ratio times the notional principal. The payout ratio depends on the recovery rate (i.e., the proportion of par that the bond trades at after default) as shown in Figure 29.2.

payout amount = payout ratio × notional principal

where:
payout ratio = 1 − recovery rate (%)

Figure 29.2: Cash Settlement on Credit Default Swap After a Credit Event

MODULE 29.2: FACTORS AFFECTING CDS PRICING

Video covering this content is available online.

LOS 29.c: Explain the principles underlying and factors that influence the market's pricing of CDS.

The factors that influence the pricing of CDS (i.e., CDS spread) include the probability of default, the loss given default, and the coupon rate on the swap. The CDS spread is higher for higher probability of default and for higher loss given default. For this discussion, we will focus on single-name CDS. However, the principles apply to index CDS as well.

Earlier, we showed the calculation of CVA, which is the present value of expected loss. CVA, therefore, would be the best estimate of the value of the hedging instrument to cover the risk of the protection seller.

Probability of default (POD) is the likelihood of default by the reference entity in a given year. However, because the CDS typically covers a multi-year horizon, the probability of default is not constant; the probability of default usually increases over time. Hence, the probability of default in any given year assumes that no default has occurred in the preceding years. We call the probability of default *given that it has not already occurred* the **conditional probability of default** or **hazard rate**. The credit risk of a reference obligation and hence the cost of protection is proportional to the hazard rate. The POD in any year is equal to the probability of survival at the end of the prior year multiplied by the hazard rate of that year.

$$POD^t = PS^{(t-1)} \times \text{hazard rate}^t$$

EXAMPLE: Hazard rate

Consider a five-year senior CDS on Xeon Corp. Xeon's hazard rate is 2% and increases by 1% per year.

Compute the survival rate in five years.

Answer:

The hazard rates for the five years are: 2%, 3%, 4%, 5%, and 6%.

survival rate in five years = (1 − 0.02)(1 − 0.03)(1 − 0.04)(1 − 0.05)(1 − 0.06) = 0.815 or 81.5%

For a single-period CDS, ignoring the time value of money, we can estimate the CDS premium as:

$$\text{CDS spread} \approx (1 - RR) \times POD$$

For example, if the recovery rate is 50% and POD is 3%, CDS spread = 0.5 × 0.03 = 0.015 or 150 bps.

The cash payments made by the protection buyer on the CDS (i.e., the coupon payments) cease when there is a default (i.e., when the CDS terminates). Hence, the expected value of the coupon payments also depends on the hazard rate.

The payments made by the protection buyer to the seller are the **premium leg**. On the other side of the contract, the protection seller must make a payment to the protection buyer in case of a default; these contingent payments make up the **protection leg**.

The difference between the present value of the premium leg and the present value of the protection leg determines the upfront payment.

$$\text{upfront payment (by protection buyer)} = PV(\text{protection leg}) - PV(\text{premium leg})$$

We can approximate the upfront premium as the difference between the CDS spread and the CDS coupon rate, multiplied by the duration of the CDS. Again, the CDS spread is the compensation for bearing the credit risk of the reference obligation.

PROFESSOR'S NOTE

Be careful here! The formula uses duration of the CDS and not the duration of the reference obligation.

$$\text{upfront premium \%} \approx (\text{CDS spread} - \text{CDS coupon}) \times \text{duration}$$

which means that:

$$\text{CDS spread} = \frac{\text{upfront premium \%}}{\text{duration}} + \text{CDS coupon}$$

We can also quote the **CDS price** as:

$$\text{price of CDS (per \$100 notional)} \approx \$100 - \text{upfront premium (\%)}$$

EXAMPLE: Upfront premium and price of CDS

Aki Mutaro, bond portfolio manager for a regional bank, is considering buying protection on one of the bank's high-yield holdings: Alpha, Inc., bonds. Ten-year CDS on Alpha bonds have a coupon rate of 5% while the 10-year Alpha CDS spread is 3.5%. The duration of the CDS is 7.

Calculate the approximate upfront premium and price of a 10-year Alpha Inc., CDS.

Answer:

$$\begin{aligned}\text{upfront premium \%} &\approx (\text{CDS spread} - \text{CDS coupon}) \times \text{duration} \\ &= (3.5\% - 5.0\%) \times 7 = -10.5\%\end{aligned}$$

Hence, the protection seller would pay (approximately) 10.5% of the notional to the protection buyer upfront because the CDS coupon is higher than the credit spread.

CDS price = 100 – (–10.5) = $110.50 per $100 notional

VALUATION AFTER INCEPTION OF CDS

At inception of a CDS, the CDS spread (and the upfront premium) is computed based on the credit quality of the reference entity. After inception, the credit quality of the reference entity (or the credit risk premium in the overall market) may change. This will lead to the underlying CDS having a nonzero value. For example, if the credit spread declines, the protection seller, having locked in a higher credit spread at initiation, would gain.

The change in value of a CDS after inception can be approximated by the change in spread multiplied by the duration of the CDS:

profit for protection buyer ≈ change in spread × duration × notional principal

or

profit for protection buyer (%) ≈ change in spread (%) × duration

Note that the protection buyer is short credit risk and hence benefits (i.e., profit is positive) when credit spreads widen.

The protection buyer (or seller) can unwind an existing CDS exposure (prior to expiration or default) by entering into an offsetting transaction. For example, a protection buyer can remove his exposure to the CDS by selling protection with the same terms as the original CDS and maturity equal to the remaining maturity on the existing CDS. The difference between the upfront premium paid and received should be (approximately) equal to the profit for the protection buyer. This process of capturing value from an in-the-money CDS exposure is called **monetizing** the gain.

MODULE 29.3: CDS USAGE

Video covering this content is available online.

LOS 29.d: Describe the use of CDS to manage credit exposures and to express views regarding changes in the shape and/or level of the credit curve.

CREDIT CURVE

The **credit curve** is the relationship between credit spreads for different bonds issued by an entity, and the bonds' maturities. The credit curve is similar to the term structure of interest rates. If the longer maturity bonds have a higher credit spread compared to shorter maturity bonds, the credit curve will be upward sloping. However, if the hazard rate is constant, the credit curve will be flat.

CDS can be used to manage credit exposures of a bond portfolio. For example, in anticipation of declining (increasing) credit spreads, a portfolio manager may increase (decrease) credit exposure in the portfolio by being a protection seller (buyer).

In a **naked CDS**, an investor with no underlying exposure purchases (or sells)protection in the CDS market. In a **long/short trade**, an investor purchases protection on one reference entity while simultaneously selling protection on another (often related) reference entity. The investor is betting that the difference in credit spreads between the two reference entities will change to the investor's advantage. This is similar to going long (protection seller exposure) in one reference entity bond and simultaneously going short (protection buyer exposure) in the other reference entity bond.

A **curve trade** is a type of long/short trade where the investor is buying and selling protection on the *same* reference entity but with a different maturity. If the investor expects that an upward-sloping credit curve on a specific corporate issuer will flatten, she may take the position of protection buyer in a short maturity CDS and the position of protection seller in a long maturity CDS.

An investor concerned about the credit risk of an issuer in the near term while being more confident of the long-term prospects of the issuer might buy protection in the short-term CDS and offset the premium cost by selling protection in the long-term CDS. An investor who believes that the short-term outlook for the reference entity is better than the long-term outlook can use a curve-steepening trade; buying protection in a long-term CDS and selling protection in a short-term CDS. The investor will profit if the credit curve steepens; that is, if long-term credit risk increases relative to short-term credit risk. Conversely, an investor who is bearish about the reference entity's prospects in the short term will enter into a curve-flattening trade.

LOS 29.e: Describe the use of CDS to take advantage of valuation disparities among separate markets, such as bonds, loans, equities, and equity-linked instruments.

USES OF CDS

Earning arbitrage profits is another motivation for trading in the CDS market. Differences in pricing between asset and derivative markets, or differences in pricing of different products in the market, may offer potential arbitrage profits.

A **basis trade** is an attempt to exploit the difference in credit spreads between bond markets and the CDS market. Basis trades rely on the idea that such mispricing will be temporary and that disparity should eventually disappear after it is recognized. For example, if a specific bond is trading at a credit spread of 4% over MRR in the

bond market but the CDS spread on the same bond is 3%, a trader can profit by buying the bond and taking the protection buyer position in the CDS market. If the expected convergence occurs, the trader will make a profit.

Another arbitrage transaction involves buying and selling debt instruments issued by the same entity based on which instruments the CDS market suggests to be undervalued or overvalued.

In a leveraged buyout (LBO), the firm will issue a great amount of debt in order to repurchase all of the company's publicly traded equity. This additional debt will increase the CDS spread because default is now more likely. An investor who anticipates an LBO might purchase both the stock and CDS protection, both of which will increase in value when the LBO eventually occurs.

In the case of an index CDS, the value of the index should be equal to the sum of the values of the index components. An arbitrage transaction is possible if the credit risk of the index constituents is priced differently than the index CDS spread.

Collateralized debt obligations (CDO) are claims against a portfolio of debt securities. A synthetic CDO has similar credit risk exposure to that of a cash CDO but is assembled using CDS rather than debt securities. If the synthetic CDO can be created at a cost lower than that of the cash CDO, investors can buy the synthetic CDO and sell the cash CDO, engaging in a profitable arbitrage.

MODULE QUIZ 29.1, 29.2, 29.3

Use the following information to answer Questions 1 through 6.

Jamshed Banaji, CFA, manages a $400 million bond portfolio for a large public pension fund. Banaji is concerned about volatility in the credit markets and expects credit spreads to widen in the short-term but revert back to current levels over the long-term.

Banaji has flagged two of his holdings for further scrutiny: IDG Corp. and Zeta Corp.

The portfolio currently has $10 million par value of 6% 10-year senior unsecured IDG Corp. bonds. Because he is concerned about IDG's credit risk, Banaji enters into a credit default swap as a protection buyer. Banaji selects a five-year senior CDS for IDG with a coupon rate of 5% and a duration of 4. IDG bonds have a yield-to-maturity of 6.5%. The MRR yield curve is flat at 2%.

Banaji is also concerned about the Zeta Corp. bonds that he holds. Zeta's management is planning to pursue a recapitalization plan that involves a large stock buyback program financed by new debt.

1. The *most* appropriate strategy for Banaji, given his expectation about changing credit spreads, is a:
   A. credit curve trade; selling protection in the short-term and purchasing protection in the long-term.
   B. credit curve trade; buying protection in the short-term and selling protection in the long-term.
   C. CDS trade; buying protection in the short-term only.
2. At inception of the CDS for IDG bonds, Banaji is *most likely* to:
   A. receive a premium of $200,000.
   B. pay a premium of $300,000.
   C. receive a premium of $400,000.
3. For this question only, suppose that six months after the inception of the swap, IDG declares bankruptcy. Figure 1 shows the market prices of IDG bonds after the

company files for bankruptcy.

Figure 1: Market Price of IDG Bonds Post Bankruptcy Filing

Description	Market Price (%) of Par
9.5-year 6% senior unsecured	45% of par
5-year 5% senior unsecured	40% of par
5-year 6% subordinated unsecured	30% of par

If Banaji has a choice of settlement procedure, he is *most likely* to choose:

A. physical settlement.

B. cash settlement and the payoff would be $6 million.

C. cash settlement and the payoff would be $7 million.

4. Which of the following statements about hazard rate is *most accurate*? Hazard rate:

   A. is the probability of default given that default has already occurred in a previous period.

   B. affects both the premium leg as well as the protection leg in a CDS.

   C. is higher for higher loss given default.

5. The *most appropriate* strategy for Banaji to follow in regard to Zeta Corp. would be to buy Zeta Corp.:

   A. stock and buy CDS protection on Zeta Corp. bonds.

   B. bonds and sell CDS protection on Zeta Corp. bonds.

   C. stock and sell CDS protection on Zeta Corp. bonds.

6. The statement "credit spreads are positively related to loss given default and to hazard rate" is:

   A. correct.

   B. correct regarding loss given default but incorrect regarding hazard rate.

   C. correct regarding hazard rate but incorrect regarding loss given default.

KEY CONCEPTS

LOS 29.a

A credit default swap (CDS) is essentially an insurance contract wherein upon occurrence of a credit event, the credit protection buyer gets compensated by the credit protection seller. To obtain this coverage, the protection buyer pays the seller a premium called the CDS spread. In the case of a single-name CDS, the reference obligation is the fixed income security on which the swap is written. An index CDS covers an equally-weighted combination of borrowers.

LOS 29.b

A default is defined as occurrence of a credit event. Common types of credit events specified in CDS agreements include bankruptcy, failure to pay, and restructuring.

When there is a credit event, the swap will be settled in cash or by physical delivery.

LOS 29.c

The factors that influence the pricing of CDS (i.e., CDS spread) include the probability of default, the loss given default, and the coupon rate on the swap. The CDS spread is higher for a higher probability of default and for a higher loss given default. The conditional probability of default (i.e., the probability of default given that default has not already occurred) is called the hazard rate.

$$\text{(expected loss)}_t = \text{(hazard rate)}_t \times \text{(loss given default)}_t$$

The upfront premium on a CDS can be computed as:

$$\text{upfront payment (by protection buyer)} = \text{PV(protection leg)} - \text{PV(premium leg)}$$

Or approximately:

$$\text{upfront premium} \approx \text{(CDS spread} - \text{CDS coupon)} \times \text{duration}$$

The change in value for a CDS after inception can be approximated by the change in spread multiplied by the duration of the CDS.

$$\text{profit for protection buyer} \approx \text{change in spread} \times \text{duration} \times \text{notional principal}$$

$$\text{profit for protection buyer (\%)} \approx \text{change in spread (\%)} \times \text{duration}$$

LOS 29.d

In a naked CDS, an investor with no exposure to the underlying purchases protection in the CDS market.

In a long/short trade, an investor purchases protection on one reference entity while selling protection on another reference entity.

A curve trade is a type of long/short trade where the investor is buying and selling protection on the same reference entity but with different maturities. An investor who believes the short-term outlook for the reference entity is better than the long-term outlook can use a curve-steepening trade (buying protection in a long-term CDS and selling protection in a short-term CDS) to profit if the credit curve steepens. Conversely, an investor who is bearish about the reference entity's prospects in the short term will enter into a curve-flattening trade.

LOS 29.e

A basis trade is an attempt to exploit the difference in credit spreads between bond markets and the CDS market. Basis trades rely on the idea that such mispricings will be temporary and that disparity should eventually disappear after it is recognized.

If a synthetic CDO can be created at a cost lower than that of the equivalent cash CDO, investors can buy the synthetic CDO and sell the cash CDO, producing a profitable arbitrage.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 29.1, 29.2, 29.3

1. **B** Banaji expects credit spreads to widen in the short-term; therefore, the appropriate strategy is to buy short-term CDS protection. Similarly, long-term credit spreads are expected to revert back to current levels (narrow) and hence Banaji can sell protection in the long-term CDS. Buying protection only would cost more money (the protection buyer premium is not offset by premium income from selling protection) and does not use Banaji's entire information set and, therefore, is not most appropriate. (Module 29.2, LOS 29.c)
2. **A** credit spread on IDG bonds = yield − MRR = 6.5% − 2% = 4.5%

upfront premium (paid by protection buyer) ≈ (CDS spread – CDS coupon) × duration × notional principal

= (0.045 – 0.05) × 4 × $10 million = –$200,000

Because the computed value is negative, $200,000 would be received by Banaji as the protection buyer. (Module 29.2, LOS 29.c)

3. **B** The CDS in the question is a senior CDS, hence the reference obligation is a senior unsecured bond. The payoff on the CDS is based on the CTD with same seniority as reference obligation. From the three choices given, the five-year 5% senior unsecured is the cheapest to deliver. Hence, the payoff will be notional principal – market value of the CTD = $10 million – $4 million = $6 million.

   Note that physical settlement would not be advantageous to Banaji; Figure 1 indicates that the IDG bonds that Banaji is currently holding have a market value of $4.5 million, so the implied payoff of physically delivering these bonds in exchange for $10 million would be only $5.5 million ($10 million – $4.5 million). (Module 29.2, LOS 29.c)

4. **B** Hazard rate is the conditional probability of default given that default has not occurred in previous periods. The hazard rate affects the protection leg: the higher the hazard rate, the higher the expected value of payoffs made by the protection seller upon default. Hazard rate also affects the premium leg because once default occurs, the CDS ceases to exist and premium income would also cease. Loss given default depends on the recovery rate and not on hazard rate (probability of default). (Module 29.2, LOS 29.c)

5. **A** Due to leveraged recapitalization of Zeta Corp., it can be expected that the credit spread on Zeta bonds would widen leading to increased value for CDS protection buyer. Additionally, the increase in stock buyback would be expected to increase the value of Zeta stock. Banaji should purchase both the stock and CDS protection, both of which will increase in value when the LBO occurs. (Module 29.3, LOS 29.d)

6. **A** Credit spreads are positively related to hazard rates and loss given default, and negatively related to recovery rates. (Module 29.3, LOS 29.d)

Topic Quiz: Fixed Income

You have now finished the Fixed Income topic section. Please log into your Schweser online dashboard and take the Topic Quiz on this section. The Topic Quiz provides immediate feedback on how effective your study has been for this material. Questions are more exam-like than typical Module Quiz or QBank questions; a score of less than 70% indicates that your study likely needs improvement. These tests are best taken timed; allow three minutes per question.

READING 30

PRICING AND VALUATION OF FORWARD COMMITMENTS

EXAM FOCUS

This topic review covers the calculation of price and value for forward and future contracts, specifically equity forward contracts, bond forward contracts, and forward (interest) rate agreements. This topic review also covers pricing and valuation of different swaps, including interest rate swaps, equity swaps, and currency swaps. Be able to structure profitable arbitrage transactions should the forward price differs from the no-arbitrage price. There are many formulas in this reading. However, once you realize the fundamental principles of no-arbitrage pricing, the formulas will start becoming somewhat less intimidating. There is a lot of testable material from this seven-LOS reading!

MODULE 30.1: PRICING AND VALUATION CONCEPTS

Video covering this content is available online.

Warm-Up: Forward Contracts

The party to the forward contract that agrees to buy the financial or physical asset has a **long forward position** and is called the *long*. The party to the forward contract that agrees to sell/deliver the asset has a **short forward position** and is called the *short*.

We will illustrate the basic forward contract mechanics through an example based on the purchase and sale of a Treasury bill. Note that while forward contracts on T-bills are usually quoted in terms of a discount percentage from face value, we use dollar prices here to make the example easy to follow.

Consider a contract under which Party A agrees to buy a $1,000 face value 90-day Treasury bill from Party B 30 days from now at a price of $990. Party A is the long and Party B is the short. Both parties have removed uncertainty about the price they will pay or receive for the T-bill at the future date. If 30 days from now T-bills are trading at $992, the short must deliver the T-bill to the long in exchange for a $990 payment. If T-bills are trading at $988 on the future date, the long must purchase the T-bill from the short for $990, the contract price.

Typically, no money changes hands at the inception of the contract, unlike futures contracts in which each party posts an initial deposit called the **margin** as a guarantee of performance.

At any point in time, including the settlement date, the party to the forward contract with the negative value will owe money to the other side. The other side of the contract will have a positive value of equal amount. Following this example, if the T-bill price is $992 at the (future) settlement date, and the short does not deliver the T-bill for $990 as promised, the short has defaulted.

FORWARD CONTRACT PRICE DETERMINATION

The No-Arbitrage Principle

The price of a forward contract is *not* the price to purchase the contract; in fact, the parties to a forward contract typically pay nothing to enter into the contract at its inception. Here, *price refers to the forward price of the underlying*. This price may be a U.S. dollar or euro price, but it is often expressed as an interest rate or currency exchange rate. For T-bills, the price will be expressed as an annualized percentage discount from face value; for coupon bonds, it will usually be expressed as a yield to maturity; for the implicit loan in a forward rate agreement (FRA), it will be expressed as an annualized MRR; and for a currency forward, it is expressed as an exchange rate between the two currencies involved. However it is expressed, this rate, yield, discount, or dollar amount is the forward price in the contract.

The price that we wish to determine is the forward price that makes the *values* of both the long and the short positions zero at contract initiation. We will use the **no-arbitrage principle**: there should be no riskless profit to be gained by a combination of a forward contract position with positions in other assets. This principle assumes that (1) transactions costs are zero, (2) there are no restrictions on short sales or on the use of short sale proceeds, and (3) both borrowing and lending can be done in unlimited amounts at the risk-free rate of interest. This concept is so important that we'll express it in a formula:

forward price = price that prevents profitable riskless arbitrage in frictionless markets

PROFESSOR'S NOTE

We cover the first two LOS out of curriculum order for ease of exposition.

LOS 30.b: Describe the carry arbitrage model without underlying cashflows and with underlying cashflows.

A Simple Version of the Cost-of-Carry Model

In order to explain the no-arbitrage condition as it applies to the determination of forward prices, we will first consider a forward contract on an asset that costs nothing to store and makes no payments to its owner over the life of the forward contract. A zero-coupon (pure discount) bond meets these criteria. Unlike gold or wheat, it has no storage costs; unlike stocks, there are no dividend payments to consider; and unlike coupon bonds, it makes no periodic interest payments.

The general form for the calculation of the forward contract price can be stated as follows:

$$FP = S_0 \times (1 + R_f)^T$$

or

$$S_0 = \frac{FP}{(1 + R_f)^T}$$

where:
FP = forward price
S_0 = spot price at inception of the contract (t = 0)
R_f = annual risk-free rate
T = forward contract term in years

EXAMPLE: Calculating the no-arbitrage forward price

Consider a 3-month forward contract on a zero-coupon bond with a face value of $1,000 that is currently quoted at $500, and suppose that the annual risk-free rate is 6%. Determine the price of the forward contract under the no-arbitrage principle.

Answer:

$$T = \frac{3}{12} = 0.25$$
$$FP = S_0 \times (1 + R_f)^T = \$500 \times 1.06^{0.25} = \$507.34$$

Now, let's explore in more detail why $507.34 is the no-arbitrage price of the forward contract.

Cash and Carry Arbitrage When the Forward Contract Is Overpriced

Suppose the forward contract is actually trading at $510 rather than the no-arbitrage price of $507.34. A short position in the forward contract requires the delivery of this bond three months from now. The arbitrage that we examine in this case amounts to borrowing $500 at the risk-free rate of 6%, buying the bond for $500, and simultaneously taking the short position in the forward contract on the zero-coupon bond so that we are obligated to deliver the bond at the expiration of the contract for the forward price of $510.

At the settlement date, we can satisfy our obligation under the terms of the forward contract by delivering the zero-coupon bond for a payment of $510, regardless of its market value at that time. We will use the $510 payment we receive at settlement from the forward contract (the forward contract price) to repay the $500 loan. The total amount to repay the loan, since the term of the loan is three months, is:

$$\text{loan repayment} = \$500 \times (1.06)^{0.25} = \$507.34$$

The payment of $510 we receive when we deliver the bond at the forward price is greater than our loan payoff of $507.34, and we will have earned an arbitrage profit of $510 – $507.34 = $2.66. Notice that this is equal to the difference between the actual forward price and the no-arbitrage forward price. The transactions are illustrated in Figure 30.1.

Figure 30.1: Cash and Carry Arbitrage When Forward Is Overpriced

Today		Three Months From Today	
Spot price of bond	$500		
Forward price	$510		
Transaction	*Cash flow*	*Transaction*	*Cash flow*
Short forward	$0	Settle short position by delivering bond	$510.00
Buy bond	–$500		
Borrow at 6%	+$500	Repay loan	– $507.34
Total cash flow	$0	Total cash flow = arbitrage profit	+$2.66

PROFESSOR'S NOTE

Here's a couple hints to help you remember which transactions to undertake for cash and carry arbitrage: (1) always start with nothing (no cash, no securities), (2) buy underpriced assets and sell overpriced assets ("buy low, sell high"), and (3) take opposite positions in the spot and forward markets.

So, if the futures contract is overpriced, you want to take a short position in those futures (which obligates you to sell at a fixed price). Because you go short in the forward market, you take the opposite position in the spot market and buy the asset. You need money to buy the asset, so you have to borrow. Therefore, to set up a cash and carry arbitrage:

forward overpriced:

borrow money ⇒ buy (go long) the spot asset ⇒ go short the asset in the forward market

So far, the underlying instrument was a zero-coupon bond, and hence had no associated cash flows during the forward contract period. If the holder of the underlying receives cash flows (as coupon or dividends), the present value of those cash flows will be subtracted from the spot price of the underlying. We will see this later in this reading.

Reverse Cash and Carry Arbitrage When the Forward Contract Is Underpriced

Suppose the forward contract is actually trading at $502 instead of the no-arbitrage price of $507.34. We reverse the arbitrage trades from the previous case and generate an arbitrage profit as follows: We sell the bond short today for $500 and simultaneously take the long position in the forward contract, which obligates us to purchase the bond in 90 days at the forward price of $502. We invest the $500 proceeds from the short sale at the 6% annual rate for three months.

In this case, at the settlement date, we receive the investment proceeds of $507.34, accept delivery of the bond in return for a payment of $502, and close out our short position by delivering the bond we just purchased at the forward price.

The payment of $502 we make as the long position in the contract is less than the investment proceeds of $507.34, and we have earned an arbitrage profit of $507.34 – $502 = $5.34. The transactions are illustrated in Figure 30.2.

Figure 30.2: Reverse Cash and Carry Arbitrage When Forward Is Underpriced

Today		Three Months From Today	
Spot price of bond	$500		
Forward price	$502		
Transaction	*Cash flow*	*Transaction*	*Cash flow*
Long forward	$0	Settle long position by buying bond	–$502.00
Short sell bond	+$500	Deliver bond to close short position	$0.00
Invest short-sale proceeds at 6%	–$500	Receive investment proceeds	+$507.34
Total cash flow	$0	Total cash flow = arbitrage profit	+$5.34

In this case, because the forward contract is underpriced, the trades are reversed from cash and carry arbitrage. To set up the reverse cash-and-carry arbitrage:

forward underpriced:

borrow asset ⇒ short (sell) spot asset ⇒ lend money ⇒ long (buy) forward

From the calculations just listed, we can see that the no-arbitrage forward price that yields a zero *value* for both the long and short positions in the forward contract at inception is the no-arbitrage price of $507.34.

PROFESSOR'S NOTE

Day count and compounding conventions vary among different financial instruments. There are three variations used in the CFA curriculum:

- All MRR-based contracts such as FRAs, swaps, caps, floors, etc.:
  - 360 days per year and simple interest
  - Multiply "r" by days/360
- Equities, bonds, and stock options:
  - 365 days per year and periodic compound interest
  - Raise (1 + r) to an exponent of days/365
- Equity indexes:
  - 365 days per year and continuous compounding
  - Raise Euler's number "e" to an exponent of "r" times days/365

MODULE 30.2: PRICING AND VALUATION OF EQUITY FORWARDS

Video covering this content is available online.

LOS 30.a: Describe how equity forwards and futures are priced, and calculate and interpret their no-arbitrage value.

Equity Forward Contracts With Discrete Dividends

Recall that the no-arbitrage forward price in our earlier example was calculated for an asset with no periodic payments. A stock, a stock portfolio, or an equity index may have expected dividend payments over the life of the contract. In order to price such a contract, we must either adjust the spot price for the present value of the expected dividends (PVD) over the life of the contract, or adjust the forward price for the future value of the dividends (FVD) over the life of the contract. The **no-arbitrage price of an equity forward contract** in either case is:

$$\text{FP (of an equity security)} = (S_0 - \text{PVD}) \times (1 + R_f)^T$$

$$FP \text{ (of an equity security)} = [S_0 \times (1 + R_f)^T] - FVD$$

For equity contracts, use a 365-day basis for calculating *T*. For example, if it is a 60-day contract, T = 60 / 365.

EXAMPLE: Calculating the price of a forward contract on a stock

Calculate the no-arbitrage forward price for a 100-day forward on a stock that is currently priced at $30.00 and is expected to pay a dividend of $0.40 in 15 days, $0.40 in 85 days, and $0.50 in 175 days. The annual risk-free rate is 5%, and the yield curve is flat.

Answer:

Ignore the dividend in 175 days because it occurs after the maturity of the forward contract.

$$PVD = \frac{\$0.40}{1.05^{15/365}} + \frac{\$0.40}{1.05^{85/365}} = \$0.7946$$

$$FP = (\$30.00 - \$0.7946) \times 1.05^{100/365} = \$29.60$$

The timeline of cash flows is shown in the following figure.

Pricing a 100-Day Forward Contract on Dividend-Paying Stock

To calculate the value of the long position in a **forward contract on a dividend-paying stock**, we make the adjustment for the present value of the remaining expected discrete dividends at time *t* (PVD_t) to get:

$$V_t(\text{long position}) = [S_t - PVD_t] - \left[\frac{FP}{(1 + R_f)^{(T-t)}}\right]$$

PROFESSOR'S NOTE

This formula still looks like the standard spot price minus present value of forward price. However, now the "spot price" has been adjusted by subtracting out the present value of the dividends because the long position in the forward contract does not receive the dividends paid on the underlying stock. So, now think adjusted spot price less present value of forward price.

If given the current forward price (FP_t) on the same underlying and with the same maturity:

$$V_t(\text{long position}) = \left[\frac{FP_t - FP}{(1 + R_f)^{T-t}}\right]$$

EXAMPLE: Calculating the value of an equity forward contract on a stock

After 60 days, the value of the stock in the previous example is \$36.00. Calculate the value of the equity forward contract on the stock to the long position, assuming the risk-free rate is still 5% and the yield curve is flat.

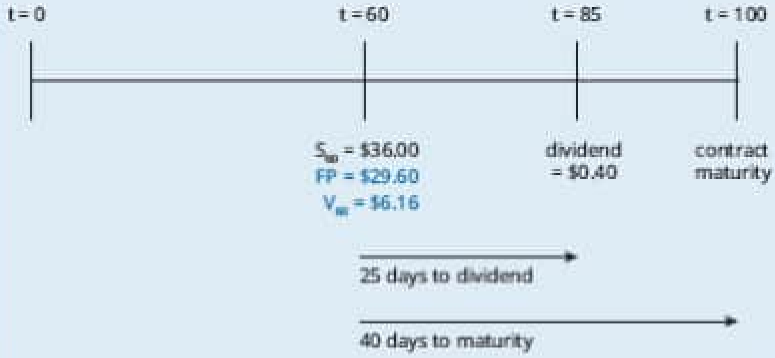
Answer:

There's only one dividend remaining (in 25 days) before the contract matures (in 40 days) as shown here, so:

$$PVD_{60} = \frac{\$0.40}{1.05^{25/365}} = \$0.3987$$

$$V_{60}(\text{long position}) = \$36.00 - \$0.3987 - \left[\frac{\$29.60}{1.05^{40/365}}\right] = \$6.16$$

Valuing a 100-Day Forward Contract After 60 Days



Equity Forward Contracts With Continuous Dividends

To calculate the price of an **equity index forward contract**, rather than take the present value of each dividend on (possibly) hundreds of stocks, we can make the calculation as if the dividends are paid continuously (rather than at discrete times) at the dividend yield rate on the index. Using continuous time discounting, we can calculate the no-arbitrage forward price as:

$$FP(\text{on an equity index}) = S_0 \times e^{(R_f^c - \delta^c)\times T} = \left(S_0 \times e^{-\delta^c \times T}\right) \times e^{R_f^c \times T}$$

where:

R_f^c = continuously compounded risk-free rate

δ^c = continuously compounded dividend yield

PROFESSOR'S NOTE

The relationship between the periodically compounded risk-free rate R_f and the continuously compounded rate R_f^c is $R_f^c = \ln(1 + R_f)$ For example, 5% compounded annually is equal to ln(1.05) = 0.04879 = 4.879% compounded continuously. The 2-year 5% future value factor can then be calculated as either $1.05^2 = 1.1025$ or $e^{0.04879\times 2} = 1.1025$.

EXAMPLE: Calculating the price of a forward contract on an equity index

The value of the S&P 500 Index is 1,140. The continuously compounded risk-free rate is 4.6% and the continuous dividend yield is 2.1%. Calculate the no-arbitrage price of a 140-day forward contract on the index.

Answer:

$$FP = 1{,}140 \times e^{(0.046 - 0.021)\times(140/365)} = 1{,}151$$

On a TI BA II PLUS calculator, use the following keystrokes:

0.046[–]0.021[=][×]140[÷]365[=][2nd][LN][×]1140[=]

MODULE QUIZ 30.1, 30.2

1. A stock is currently priced at $30 and is expected to pay a dividend of $0.30 20 days and 65 days from now. The contract price for a 60-day forward contract when the interest rate is 5% is *closest* to:
   A. $29.46.
   B. $29.70.
   C. $29.94.
2. After 37 days, the stock in Question 1 is priced at $21, and the risk-free rate is still 5%. The value of the forward contract on the stock to the short position is:
   A. –$8.85.
   B. +$8.85.
   C. +$9.00.
3. The forward price of a 200-day stock index futures contract when the spot index is 540, the continuous dividend yield is 1.8%, and the continuously compounded risk-free rate is 7% (with a flat yield curve) is *closest* to:
   A. 545.72.
   B. 555.61.
   C. 568.08.
4. An analyst who mistakenly ignores the dividends when valuing a short position in a forward contract on a stock that pays dividends will *most likely*:
   A. overvalue the position by the present value of the dividends.
   B. undervalue the position by the present value of the dividends.
   C. overvalue the position by the future value of the dividends.
5. A portfolio manager owns Macrogrow, Inc., which is currently trading at $35 per share. She plans to sell the stock in 120 days, but is concerned about a possible price decline. She decides to take a short position in a 120-day forward contract on the stock. The stock will pay a $0.50 per share dividend in 35 days and $0.50 again in 125 days. The risk-free rate is 4%. The value of the trader's position in the forward contract in 45 days, assuming in 45 days the stock price is $27.50 and the risk-free rate has not changed, is *closest* to:
   A. $7.17.
   B. $7.50.
   C. $7.92.

MODULE 30.3: PRICING AND VALUATION OF FIXED INCOME FORWARDS

Video covering this content is available online.

LOS 30.d: Describe how fixed-income forwards and futures are priced, and calculate and interpret their no-arbitrage value.

Forwards and Futures on Fixed Income Securities

In order to calculate the no-arbitrage **forward price on a coupon-paying bond**, we can use the same formula as we used for a dividend-paying stock or portfolio, simply substituting the present value of the expected coupon payments (PVC) *over the life of the contract* for PVD, or the future value of the coupon payments (FVC) for FVD, to get the following formulas:

$$\text{FP(on a fixed income security)} = (S_0 - \text{PVC}) \times (1 + R_f)^T$$

or

$$= S_0 \times (1 + R_f)^T - \text{FVC}$$

The value of the forward contract prior to expiration is as follows:

$$V_t(\text{long position}) = [S_t - \text{PVC}_t] - \left[\frac{\text{FP}}{(1 + R_f)^{(T-t)}}\right]$$

If given the current forward price (FP_t) on the same underlying and with the same maturity:

$$V_t(\text{long position}) = \left[\frac{\text{FP}_t - \text{FP}}{(1 + R_f)^{(T-t)}}\right]$$

In our examples, we assume that the spot price of the underlying coupon-paying bond includes accrued interest. For fixed income contracts, use a 365-day basis to calculate *T* if the contract maturity is given in days.

EXAMPLE: Calculating the price of a forward on a fixed income security

Calculate the price of a 250-day forward contract on a 7% U.S. Treasury bond with a spot price of $1,050 (including accrued interest) that has just paid a coupon and will make another coupon payment in 182 days. The annual risk-free rate is 6%.

Answer:

Remember that U.S. Treasury bonds make semiannual coupon payments, so:

$$C = \frac{\$1{,}000 \times 0.07}{2} = \$35.00$$

$$\text{PVC} = \frac{\$35.00}{1.06^{182/365}} = \$34.00$$

The forward price of the contract is therefore:

$$\text{FP (on a fixed income security)} = (\$1{,}050 - \$34.00) \times 1.06^{250/365} = \$1{,}057.37$$

EXAMPLE: Calculating the value of a forward on a fixed income security

After 100 days, the value of the bond in the previous example is $1,090. Calculate the value of the forward contract on the bond to the long position, assuming the risk-free rate is 6.0%.

Answer:

There is only one coupon remaining (in 82 days) before the contract matures (in 150 days), so:

$$PVC = \frac{\$35.00}{1.06^{82/365}} = \$34.54$$

$$V_{100} \text{ (long position)} = \$1{,}090 - \$34.54 - \left(\frac{\$1{,}057.37}{1.06^{150/365}}\right) = \$23.11$$

If given the forward price directly, the value (to the long party) is simply the present value of the difference between the current forward price and the original, locked-in forward price.

EXAMPLE: Value of a fixed-income forward contract after inception

Louise Michelle entered into a 250-day forward contract on a 7% U.S. Treasury bond at a forward price of $1057.37. 100 days later, the forward price has changed to $1081.04. The risk-free rate is 6%.

Calculate the value to Michelle of the forward contract.

Answer:

$$\text{value} = [1081.04 - 1057.37] / (1.06)^{(150/365)} = \$23.11$$

Bond futures contracts often allow the short an option to deliver any of several bonds, which will satisfy the delivery terms of the contract. This is called a delivery option and is valuable to the short. Each bond is given a conversion factor that is used to adjust the long's payment at delivery so the more valuable bonds receive a larger payment. These factors are multipliers for the futures price at settlement. The long pays the futures price at expiration, multiplied by the **conversion factor (CF)**.

Bond prices in some countries are quoted as **clean prices**. At settlement, the buyer actually pays the clean price plus an **accrued interest**, or the **full price**.

$$\text{accrued interest} = \left(\frac{\text{days since last coupon payment}}{\text{days between coupon payments}}\right) \times \text{coupon amount}$$

$$\text{full price} = \text{clean price} + \text{accrued interest} = \text{clean price} + AI_0$$

The futures price can then be calculated as:

$$FP = \left[(\text{full price})\left(1 + R_f\right)^T - FVC - AI_T\right]$$

where:
AI_T = the accrued interest at maturity of the futures contract

The quoted futures price adjusts this price based on the conversion factor (CF) as follows:

$$QFP = FP/CF = \left[(\text{full price})(1+R_f)^T - FVC - AI_T\right]\left(\frac{1}{CF}\right)$$

EXAMPLE: Calculating the price of a Treasury bond futures contract

Suppose that you need to calculate the quoted futures price of a 1.2-year Treasury bond futures contract. The cheapest-to-deliver bond is a 7% T-bond with exactly 10 years remaining to maturity and a quoted price of $1,040 with a conversion

factor of 1.13. There is currently no accrued interest because the bond has just paid a coupon. The annual risk-free rate is 5%. The accrued interest on the bond at maturity of the futures contract will be $14.

Answer:

The full price of the bond = $1,040 quoted price + $0 accrued interest = $1,040. The semiannual coupon on a single, $1,000 face-value 7% bond is $35. A bondholder will receive one payment 0.5 years from now (when there are 0.7 years left to maturity of the futures contract) and one payment 1 year from now (when there are 0.2 years until maturity). The future value of these coupons at the end of 1.2 years (the expiration date) is:

$$\text{FVC} = (\$35 \times 1.05^{0.7}) + (\$35 \times 1.05^{0.2}) = \$71.56$$

The quoted futures price is then:

$$\text{QFP} = [(\$1{,}040 \times 1.05^{1.2}) - \$71.56 - \$14]\left(\frac{1}{1.13}\right) = \$900.13$$

MODULE QUIZ 30.3

1. A 6% Treasury bond is trading at $1,044 (including accrued interest) per $1,000 of face value. It will make a coupon payment 98 days from now. The yield curve is flat at 5% over the next 150 days. The forward price per $1,000 of face value for a 120-day forward contract, is *closest* to:
   A. $1,014.52.
   B. $1,030.79.
   C. $1,037.13.

MODULE 30.4: PRICING FORWARD RATE AGREEMENTS

Video covering this content is available online.

LOS 30.c: Describe how interest rate forwards and futures are priced, and calculate and interpret their no-arbitrage value.

Warm-Up: MRR-Based Loans and Forward Rate Agreements

Eurodollar deposit is the term for deposits in large banks outside the United States denominated in U.S. dollars. The lending rate on dollar-denominated loans between banks is based on the **secured overnight funding rate (SOFR)**, hereafter referred to generically as the MRR ("market reference rate"). It is quoted as an annualized rate based on a 360-day year. In contrast to T-bill discount yields, MRR is an add-on rate, like a yield quote on a short-term certificate of deposit.

EXAMPLE: MRR-based loans

Compute the amount that must be repaid on a $1 million loan for 30 days if 30-day MRR is quoted at 6%.

Answer:

The add-on interest is calculated as \$1 million × 0.06 × (30 / 360) = \$5,000. The borrower would repay \$1,000,000 + \$5,000 = \$1,005,000 at the end of 30 days.

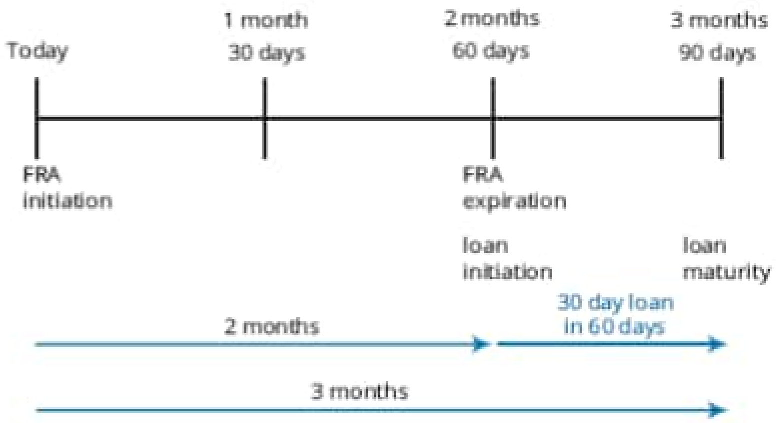
The long position in a **forward rate agreement (FRA)** is the party that is effectively borrowing money (long the loan, with the contract price being the interest rate on the loan). If the floating rate at contract expiration is above the rate specified in the forward agreement, the long position in the contract can be viewed as the right to borrow at below-market rates, and the long will receive a payment. If the floating rate at the expiration date is below the rate specified in the forward agreement, the short will receive a cash payment from the long. (The right to lend at *above*-market rates has a positive value.)

PROFESSOR'S NOTE

We say "can be viewed as" because an FRA is settled in cash, so there is no requirement to lend or borrow the amount stated in the contract. For this reason, the creditworthiness of the long position is not a factor in the determination of the interest rate on an FRA. However, to understand the pricing and calculation of value for an FRA, viewing the contract as a commitment to lend or borrow at a certain interest rate at a future date is helpful.

The notation for FRAs is unique. There are two numbers associated with an FRA: the number of months until the contract expires, and the number of months until the underlying loan is settled. The difference between these two is the maturity of the underlying loan. For example, a 2 × 3 FRA is a contract that expires in two months (60 days), and the underlying loan is settled in three months (90 days). The underlying rate is 1-month (30-day) MRR on a 30-day loan in 60 days. See Figure 30.3.

Figure 30.3: Illustration of a 2 × 3 FRA



Pricing FRAs

There are three important things to remember about FRAs when we're pricing and valuing them:

1. MRR rates in the Eurodollar market are add-on rates and are always quoted on a 30/360 day basis in annual terms. For example, if the MRR quote on a 30-day loan is 6%, the actual unannualized monthly rate is 6% × (30/360) = 0.5%.

2. The long position in an FRA is, in effect, long the rate and benefits when the rate increases.
3. Although the interest on the underlying loan won't be paid until the end of the loan (e.g., in three months in Figure 30.3, the payoff on the FRA occurs at the expiration of the FRA (e.g., in two months). Therefore, the payoff on the FRA is the present value of the interest savings on the loan (e.g., discounted one month in Figure 30.3).

The forward "price" in an FRA is actually a forward interest rate. The calculation of a forward interest rate is presented in Level I as the computation of forward rates from spot rates. We will illustrate this calculation with an example.

EXAMPLE: Calculating the price of an FRA

Calculate the price of a 1 × 4 FRA (i.e., a 90-day loan, 30 days from now). The current 30-day MRR is 4% and 120-day MRR is 5%.

Answer:

The actual (unannualized) rate on the 30-day loan is:

$$\text{R30} = 4\% \times \frac{30}{360} = 0.333\%$$

The actual (unannualized) rate on the 120-day loan is:

$$\text{R120} = 5\% \times \frac{120}{360} = 1.667\%$$

We wish to calculate the actual rate on a 90-day loan from day 30 to day 120:

$$\text{price of } 1 \times 4 \text{ FRA} = \frac{1 + \text{R120}}{1 + \text{R30}} - 1 = \frac{1.01667}{1.00333} - 1 = 1.33\%$$

We can annualize this rate as:

$$1.33\% \times \frac{360}{90} = 0.0532 = 5.32\%$$

This is the no-arbitrage forward rate—the forward rate that will make the values of the long and the short positions in the FRA both zero at the initiation of the contract.

The time line is shown in the following figure.

Pricing a 1 × 4 FRA

MODULE QUIZ 30.4

1. The contract rate (annualized) for a 3 × 5 FRA if the current 60-day rate is 4%, the current 90-day rate is 5%, and the current 150-day rate is 6%, is *closest* to:
   A. 6.0%.
   B. 6.9%.
   C. 7.4%.
2. The CFO of Yellow River Company wishes to hedge borrowing costs on a 90-day loan that starts in 90 days. The current term structure indicates the following: 30-day MRR is 4.5%, 90-day MRR is 4.7%, and 180-day MRR is 4.9%.

   The appropriate hedging position to take, and the corresponding fair market rate, would *best* be described as:
   A. long 3 × 3 FRA at a rate of 4.48%.
   B. long 3 × 6 FRA at a rate of 4.48%.
   C. long 3 × 6 FRA at a rate of 5.02%.

MODULE 30.5: VALUATION OF FORWARD RATE AGREEMENTS

Video covering this content is available online.

Valuing an FRA at Maturity

To understand the calculation of the value of the FRA *after the initiation of the contract*, recall that in the previous example the long in the FRA has the "right" to borrow money 30 days from inception for a period of 90 days at the forward rate. If interest rates increase (specifically the 90-day forward contract rate), the long will profit as the contract has fixed a borrowing rate below the now-current market rate. These "savings" will come at the end of the loan term, so to value the FRA we need to take the present value of these savings. An example incorporating this fact will illustrate the cash settlement value of an FRA at expiration.

EXAMPLE: Calculating the value of an FRA at maturity (i.e., cash payment at settlement)

Continuing the prior example for a 1 × 4 FRA, assume a notional principal of $1 million. Suppose that, at contract expiration, the 90-day rate has increased to 6%, which is above the contract rate of 5.32%. Calculate the value of the FRA at maturity, which is equal to the cash payment at settlement.

Answer:

The interest savings at the end of the loan term (compared to the market rate of 6%) will be:

$$\left[\left(0.0600 \times \frac{90}{360}\right) - \left(0.0532 \times \frac{90}{360}\right)\right] \times \$1{,}000{,}000 = \$1{,}700$$

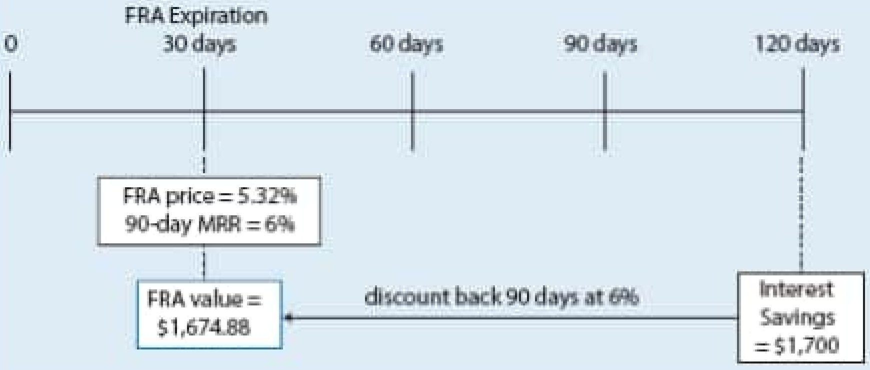
The present value of this amount at the FRA settlement date (90 days prior to the end of the loan term) discounted at the current rate of 6% is:

$$\frac{\$1{,}700}{1 + \left(0.06 \times \frac{90}{360}\right)} = \$1{,}674.88$$

This will be the cash settlement payment from the short to the long at the expiration of the contract. Note that we have discounted the savings in interest at

the end of the loan term by the *market* rate of 6% that prevails at the contract settlement date for a 90-day term, as shown in the following figure.

Valuing a 1 × 4 FRA at Maturity



Valuing an FRA Before Maturity

To value an FRA *before* the settlement date, we need to know the number of days that have passed since the initiation of the contract. For example, let's suppose we want to value the same 1 × 4 FRA 10 days after initiation. Originally, it was a 1 × 4 FRA, which means the FRA now expires in 20 days. The calculation of the "savings" on the loan will be the same as in our previous example, except that we need to use the "new" FRA price that would be quoted on a contract covering the same period as the original "loan." In this case the "new" FRA price is the now-current market forward rate for a 90-day loan made at the settlement date (20 days in the future). Also, we need to discount the interest savings implicit in the FRA back 110 days (i.e., an extra 20 days) instead of 90 days as we did for the value at the settlement date.

EXAMPLE: Calculating value of an FRA before settlement

Value a 5.32% 1 × 4 FRA with a principal amount of $1 million 10 days after initiation if 110-day MRR is 5.9% and 20-day MRR is 5.7%.

Answer:

Step 1: Find the "new" FRA price on a 90-day loan 20 days from today. This is the current 90-day forward rate at the settlement date, 20 days from now.

$$\left[\frac{1+\left(0.059\times\frac{110}{360}\right)}{1+\left(0.057\times\frac{20}{360}\right)}-1\right]\times\frac{360}{90} = 0.0592568$$

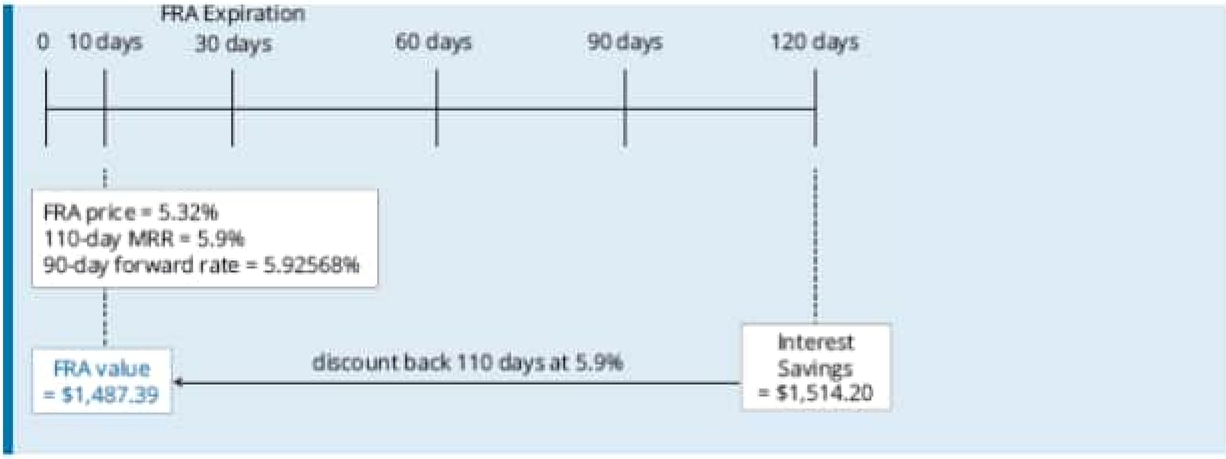
Step 2: Calculate the interest difference on a $1 million, 90-day loan made 20 days from now at the forward rate calculated previously compared to the FRA rate of 5.32%.

$$\left[\left(0.0592568\times\frac{90}{360}\right)-\left(0.0532\times\frac{90}{360}\right)\right]\times \$1{,}000{,}000 = \$1{,}514.20$$

Step 3: Discount this amount at the current 110-day rate.

$$\frac{\$1{,}514.20}{1+\left(0.059\times\frac{110}{360}\right)} = \$1{,}487.39$$

Valuing a 1 × 4 FRA Before Settlement



PROFESSOR'S NOTE

I have tried to explain these calculations in such a way that you can value an FRA at any date from initiation to settlement using basic tools that you already know. Once you understand where the value of an FRA comes from (the interest savings on a loan to be made at the settlement date) and when this value is to be received (at the end of the loan), you can calculate the present value of these savings even under somewhat stressful test conditions. Just remember that if the rate in the future is less than the FRA rate, the long is "obligated to borrow" at above-market rates and will have to make a payment to the short. If the rate is greater than the FRA rate, the long will receive a payment from the short.

Futures Contracts

Futures contracts are very much like the forward contracts except that they trade on organized exchanges. Each exchange has a clearinghouse. The clearinghouse guarantees that traders in the futures market will honor their obligations. The clearinghouse does this by splitting each trade once it is made and acting as the opposite side of each position. To safeguard the clearinghouse, the exchange requires both sides of the trade to post margin and settle their accounts on a daily basis. Thus, the margin in the futures markets is a performance guarantee.

Marking to market is the process of adjusting the margin balance in a futures account each day for the change in the value of the contract from the previous trading day, based on the settlement price.

The pricing relationship discussed earlier also generally applies to futures contracts as well (see for example, our earlier discussion on Treasury bond futures contract). Like forward contracts, futures contracts have no value at contract initiation. Unlike forward contracts, futures contracts do not accumulate value changes over the term of the contract. Since futures accounts are marked to market daily, the value after the margin deposit has been adjusted for the day's gains and losses in contract value is always zero. The futures price at any point in time is the price that makes the value of a new contract equal to zero. The value of a futures contract strays from zero only during the trading periods between the times at which the account is marked to market:

value of futures contract = current futures price − previous mark-to-market price

If the futures price increases, the value of the long position increases. The value is set back to zero by the mark-to-market at the end of the mark-to-market period.

MODULE QUIZ 30.5

1. Ina bank entered into a 3×6 FRA at a rate of 3% on a $10 million notional 30 days ago. Current MRR term structure is as follows:

Days	Rate
30	2.5%
60	2.6%
90	2.7%
120	2.9%
150	3.2%

The value of the long position in the FRA is *closest* to:

A. 14,407.
B. 14,612.
C. 57,678.

MODULE 30.6: PRICING AND VALUATION OF INTEREST RATE SWAPS

Video covering this content is available online.

LOS 30.e: Describe how interest rate swaps are priced, and calculate and interpret their no-arbitrage value.

The distinction between the price and the value of a swap is the same one we made for forward contracts. Remember that the price of a forward contract is the forward rate that yields a zero value for the contract at initiation. After contract initiation, as rates or prices change, the contract will likely have value to either the long or the short.

Consider an interest rate swap. One party agrees to pay floating (borrow at the floating rate) and receive fixed (lend at a fixed rate). At initiation of the swap, the fixed rate is selected so that the present value of the floating-rate payments is equal to the present value of the fixed-rate payments, which means the swap value is zero to both parties. This fixed rate is called the **swap rate** or the **swap fixed rate**. Determining the swap rate is equivalent to "pricing" the swap.

As short-term rates (and expected future short-term rates) change over the term of the swap, the value of the swap to one of the parties will be positive. The swap position is an asset to that party.

If market interest rates increase during the life of the swap, the swap will take on a positive value from the perspective of the fixed-rate payer. The value of the swap contract to the other party (the pay-floating side) will thus be negative.

Computing the Swap Fixed Rate

The **swap fixed rate** is derived from the MRR curve corresponding to the swap tenor. Consider a two-year, semiannual interest rate swap. The swap fixed rate

underlying this swap will be determined based on the MRR rates corresponding to the four settlement dates of this swap.

Using those four MRRs, we calculate the discount factors (Zs) for each.

$$Z = \frac{1}{\left[1 + \left(\text{MRR} \times \frac{\text{days}}{360}\right)\right]}$$

The periodic swap fixed rate SFR(periodic) can then be calculated as:

$$\text{SFR(periodic)} = \frac{1 - \text{final discount factor}}{\text{sum of discount factors}}$$

$$\text{swap fixed rate (annual)} = \text{SFR(periodic)} \times \text{number of settlement periods per year}$$

EXAMPLE: Calculating the fixed rate on a swap with quarterly payments

Annualized MRR spot rates today are:

$R_{90\text{-day}} = 0.030$

$R_{180\text{-day}} = 0.035$

$R_{270\text{-day}} = 0.040$

$R_{360\text{-day}} = 0.045$

You're analyzing a 1-year swap with quarterly payments and a notional principal amount of $5,000,000. Calculate:

- The fixed rate in percentage terms.
- The quarterly fixed payments in $.
- The first net payment.

Answer:

First calculate the discount factors. Don't forget to convert from annualized rates to per-period rates.

$$Z_{90\text{-day}} = \frac{1}{1 + \left(0.030 \times \frac{90}{360}\right)} = 0.99256$$

$$Z_{180\text{-day}} = \frac{1}{1 + \left(0.035 \times \frac{180}{360}\right)} = 0.98280$$

$$Z_{270\text{-day}} = \frac{1}{1 + \left(0.040 \times \frac{270}{360}\right)} = 0.97087$$

$$Z_{360\text{-day}} = \frac{1}{1 + \left(0.045 \times \frac{360}{360}\right)} = 0.95694$$

PROFESSOR'S NOTE

It is likely that on the exam you will be given a table of discount factors to use in pricing a swap, as in the next example. We've provided the calculations here so you know where the discount factors come from.

The quarterly fixed rate on the swap is:

$$\frac{1-0.95694}{0.99256+0.98280+0.97087+0.95694} = 0.011 = 1.1\%$$

The quarterly fixed-rate payments, assuming a notional principal of $5,000,000, are:

$5,000,000 × 0.011 = $55,000

The fixed rate on the swap in annual terms is:

$$1.1\% \times \frac{360}{90} = 4.4\%$$

We have priced a swap in which one side pays quarterly MRR and the other side pays 4.4% fixed annually (1.1% quarterly).

The first floating rate payment will be based on the 90-day rate known at inception (i.e., 3%). So the first net payment of 1.4% adjusted for quarterly time interval will be paid by the fixed-rate payer:

(0.014) ´ (90/360) ´ 5,000,000 = $17,500

Calculating the Market Value of an Interest Rate Swap

For the purpose of the exam, we are only responsible for valuation of an interest rate swap *on* settlement dates (after settlement has occurred), and not *between* settlement dates.

After the initiation of an interest rate swap, the swap will take on a positive or negative value as interest rates change. The party that is the fixed-rate payer benefits if rates increase because they are paying the older (and lower) fixed rate and receiving the newer (and higher) floating rate. Similarly, the fixed-rate receiver benefits if rates decrease because they are receiving the older (and higher) fixed rate while paying a newer (and lower) floating rate. The value is calculated as present value of the difference in payments (under the new current rate, relative to the payments under the older, locked-in rate).

$$\text{value to the payer} = \Sigma Z \times (SFR_{New} - SFR_{Old}) \times \frac{\text{days}}{360} \times \text{notional principal}$$

where:

ΣZ = the sum of discount factors associated with the remaining settlement periods

days = number of days in the settlement period

EXAMPLE: Valuing a swap on payment date

Let's continue our previous example of a one-year, quarterly settlement, $5 million notional swap with a swap fixed rate at initiation of 4.4%. Suppose that after 180 days, the MRR term structure is flat at 3.5% over the next year. Calculate the value of the swap to the fixed-rate receiver.

Answer:

Because the MRR term structure is flat at 3.5%, the new swap fixed rate must be 3.5%. At t = 180, there are two more settlement dates remaining: 90 and 180 days in the future.

$$Z_{90\text{-day}} = \frac{1}{1 + \left(0.035 \times \frac{90}{360}\right)} = 0.9913$$

$$Z_{180\text{-day}} = \frac{1}{1 + \left(0.035 \times \frac{180}{360}\right)} = 0.9828$$

$\Sigma Z = 0.9913 + 0.9828 = 1.9741$

value of the swap to the payer =

$1.9741 \times (0.035 - 0.044) \times \frac{90}{360} \times \$5,000,000 = -\$22,209$

value to the receiver = +\$22,209

MODULE QUIZ 30.6

1. Consider a one-year, quarterly settlement interest rate swap. Annualized MRR spot rates and the present value factors today are the following.

	Rate	Present Value Factor
90-day MRR	4.2%	0.98961
180-day MRR	4.8%	0.97656
270-day MRR	5.0%	0.96386
360-day MRR	5.2%	0.95057
Total		3.88060

The annualized swap rate is *closest* to:

A. 1.27%.
B. 2.54%.
C. 5.08%.

Use the following information to answer Questions 2 and 3.

Two parties enter into a 2-year fixed-for-floating interest rate swap with semiannual payments. The floating-rate payments are based on MRR. The 180-, 360-, 540-, and 720-day annualized MRR rates and present value factors are:

	Rate	Present Value Factor
180-day MRR	5.0%	0.9756
360-day MRR	6.0%	0.9434
540-day MRR	6.5%	0.9112
720-day MRR	7.0%	0.8772

2. The swap rate is *closest* to:

A. 6.62%.
B. 6.87%.
C. 7.03%.

3. After 180 days, the swap is marked-to-market when the 180-, 360-, and 540-day annualized MRRs are 4.5%, 5%, and 6%, respectively. The present value factors, respectively, are 0.9780, 0.9524, and 0.9174. What is the market value of the swap per \$1 notional principal, and which of the two counterparties (the fixed-rate payer

or the fixed-rate receiver) would make the payment to mark the swap to market?

	Market value	Payment made by
A.	\$0.01166	Fixed-rate payer
B.	\$0.04290	Fixed-rate payer
C.	\$0.01166	Fixed-rate receiver

MODULE 30.7: CURRENCY SWAPS

Video covering this content is available online.

LOS 30.f: Describe how currency swaps are priced, and calculate and interpret their no-arbitrage value.

Currency Swaps

Determining the Fixed Rate and Foreign Notional Principal

Consider two currencies, the U.S. dollar (\$) and the British pound (£), where the exchange rate is currently \$2 per £ or £0.5 per \$. The interest rates in a currency swap are simply the swap rates calculated from each country's yield curve in the relevant country's currency. Don't forget: With currency swaps, there are two yield curves and two swap fixed rates, one for each currency.

The principal amounts of the fixed-rate obligations must be adjusted for the current exchange rate. We need \$2.00 to equal £1.00, so these are the notional amounts, exchanged at the inception of the swap and returned at the termination of the swap. For example, if the notional principal amount of the \$ side of the swap is \$25 million, the £ notional principal amount will be £12.5 million.

EXAMPLE: Calculating the fixed rate and notional principal on a currency swap

In a previous example, we determined that the fixed rate on a 1-year quarterly \$5,000,000 interest rate swap, given the following set of spot MRRs, was 1.1% quarterly, or 4.4% on an annual basis.

$$R^{\$}_{90\text{-day}} = 0.030$$
$$R^{\$}_{180\text{-day}} = 0.035$$
$$R^{\$}_{270\text{-day}} = 0.040$$
$$R^{\$}_{360\text{-day}} = 0.045$$

The comparable set of £ rates are:

$$R^{£}_{90\text{-day}} = 0.04$$
$$R^{£}_{180\text{-day}} = 0.05$$
$$R^{£}_{270\text{-day}} = 0.06$$
$$R^{£}_{360\text{-day}} = 0.07$$

The current exchange rate is £0.50 per \$.

Determine the fixed rate on a 1-year £ interest rate swap. Then determine the notional £ principal amount and the quarterly cash flows on a Pay \$ fixed, receive £ fixed currency swap.

Answer:

First calculate the £ discount factors. Don't forget to convert from annualized rates to per-period rates.

$$Z^{£}_{90\text{-day}} = \frac{1}{1+\left(0.04\times\frac{90}{360}\right)} = 0.99010$$

$$Z^{£}_{180\text{-day}} = \frac{1}{1+\left(0.05\times\frac{180}{360}\right)} = 0.97561$$

$$Z^{£}_{270\text{-day}} = \frac{1}{1+\left(0.06\times\frac{270}{360}\right)} = 0.95694$$

$$Z^{£}_{360\text{-day}} = \frac{1}{1+\left(0.07\times\frac{360}{360}\right)} = 0.93458$$

The quarterly fixed rate on the £ swap is:

$$\frac{1-0.93458}{0.99010+0.97561+0.95694+0.93458} = 0.017 = 1.7\%$$

The fixed rate on the £ swap in *annual* terms is:

$$1.7\% \times \frac{360}{90} = 6.8\%$$

The notional £ principal amount of the swap would be:

$5,000,000 × £0.50 per $ = £2,500,000

At the initiation of the swap, we would exchange £2,500,000 for $5,000,000. We would pay 1.1% quarterly on the $5,000,000 notional principal ($55,000) and receive 1.7% on £2,500,000 quarterly (£42,500). At the end of one year, we would exchange the original principal amounts.

The value to any party in a currency swap is the present value of the cash flows they expect to receive minus the present value of the cash flows they are obligated to pay.

EXAMPLE: Calculating the value of a currency swap after initiation

Use the data on the $ and £ interest rate swaps in the previous examples to answer this question. After 300 days, the 60-day $ interest rate is 5.4%, the 60-day £ interest rate is 6.6%, and the exchange rate is £0.52 per $. Calculate the value of a $5,000,000 swap to the counterparty that receives $ fixed and pays £ fixed.

Answer:

Recall that the $ fixed rate was calculated as 4.4% (or 1.1% per quarter) and the £ fixed rate was calculated as 6.8% (or 1.7% per quarter). After 300 days, the only cash flows remaining are the last interest payments and the principal payments in 60 days. We want to find the present value of those cash flows, so we need the 60-day discount factors based on the current 60-day rates:

$$Z^{\$}_{60\text{-day}} = \frac{1}{1 + \left(0.054 \times \frac{60}{360}\right)} = 0.99108$$

$$Z^{£}_{60\text{-day}} = \frac{1}{1 + \left(0.066 \times \frac{60}{360}\right)} = 0.98912$$

First calculate the value after 300 days of the $ fixed payments. It is equivalent to the value of a $5,000,000 bond that matures in 60 days and makes a coupon payment in 60 days of $55,000 ($5,000,000 × 0.011). The value in $ of the $ fixed side is the present value of $5,055,000 discounted using the 60-day $ discount factor of 0.99108:

value of $ fixed side (in $) = 0.99108 × $5,055,000 = $5,009,909

Next, calculate the value after 300 days of the £ fixed payments. The value in £ of the £ fixed side is the present value of £2,542,500 discounted using the 60-day £ discount factor of 0.98912:

value of £ fixed side (in £) = 0.98912 × £2,542,500 = £2,514,838

This last step is to convert that amount into $ at the *current spot exchange rate* of £0.52 per $, to find the value of the £ fixed side in $:

value of £ fixed side (in $) = £2,514,838 / 0.52 = $4,836,227

Finally, the value of the receive $ fixed, pay £ fixed side of the swap is equal to the value of the $ fixed side minus the £ fixed side, or $5,009,909 − $4,836,227 = $173,682.

MODULE QUIZ 30.7

1. The current exchange rate is 0.70 USD/CAD. In a US$1 million fixed-for-floating currency swap, the party that is entering the swap to hedge an existing exposure to a C$-denominated fixed-rate liability will:
   A. receive $1 million at the termination of the swap.
   B. pay a fixed rate based on the yield curve in the United States.
   C. receive a fixed rate based on the yield curve in Canada.

2. A bank entered into a 1-year currency swap with quarterly payments 200 days ago by agreeing to swap $1,000,000 for €800,000. The bank agreed to pay an annual fixed rate of 5% on the €800,000 and receive a fixed rate of 4.2% on the $1,000,000. Current USD and euro MRRs and present value factors are shown in the following table.

	Rate	Present Value Factor
70-day USD MRR	4.0%	0.9923
90-day USD MRR	4.4%	0.9891
160-day USD MRR	4.8%	0.9791
180-day USD MRR	5.2%	0.9747
70-day EUR MRR	5.2%	0.9900
90-day EUR MRR	5.6%	0.9862
160-day EUR MRR	6.1%	0.9736
180-day EUR MRR	6.3%	0.9695

The current spot exchange rate is €0.75 per $1.00. The value of the swap to the bank today is *closest* to:

A. –$64,888.
B. –$42,049.
C. $42,049.

MODULE 30.8: EQUITY SWAPS

Video covering this content is available online.

LOS 30.g: Describe how equity swaps are priced, and calculate and interpret their no-arbitrage value.

Equity Swaps

To price an *N*-period pay-fixed equity swap, we can use the same formula as for a plain vanilla swap:

$$\text{SFR(periodic)} = \frac{1 - \text{final discount factor}}{\text{sum of discount factors}}$$

To value this swap, consider two scenarios:

1. Valuation on settlement date (immediately after settlement): We can continue to use the valuation formula discussed for valuation of interest rate swaps.
2. Valuation between settlement dates (for equity swaps, this is covered in the curriculum): We calculate the values of the two sides of the swap separately as shown in the following example.

EXAMPLE: Valuing a pay-fixed, receive-equity-returns swap

A $10 million principal, one-year, quarterly settlement equity swap has a fixed rate of 6.05%. The index at inception is 985. After 30 days have passed, the index stands at 996 and the term structure of MRR is 6%, 6.5%, 7%, and 7.5% for terms of 60, 150, 240, and 330 days, respectively. The respective discount factors are 0.99010, 0.97363, 0.95541, and 0.93567. Calculate the value of the swap to the fixed-rate payer on Day 30.

Answer:

Value of the fixed side (per $1 notional) = present value of coupons + PV of principal

= (0.0605/4) (0.99010 + 0.97363 + 0.95541 + 0.93567) + (0.93567) (1)

= 0.9940

Value of the fixed-rate side of the swap ($10 million notional) = 0.9940 × $10 million = $9,940,000

The value of $10,000,000 invested in the index after 30 days is:

$10,000,000 × (996/985) = $10,111,675

From the standpoint of the fixed-rate payer, the value of the swap after 30 days is:

$10,111,675 – $9,940,000 = $171,675.

A swap of returns on two different stocks can be viewed as buying one stock (receiving the returns) and shorting an equal value of a different stock (paying the returns). There is no "pricing" at swap initiation, and we can value the swap at any point in time by taking the difference in returns (since the last payment date) times the notional principal.

EXAMPLE: Valuing a "one-equity-return-for-another" swap

An investor is the Stock A returns payer (and Stock B returns receiver) in a $1 million quarterly-pay swap. After one month, Stock A is up 1.3% and Stock B is down 0.8%. Calculate the value of the swap to the investor.

Answer:

The investor pays the Stock A returns and receives Stock B returns. However, the Stock B returns are negative, so he pays those as well:

value of swap = (–0.013 – 0.008) × $1,000,000 = –$21,000

MODULE QUIZ 30.8

1. A bank entered into a $5,000,000, 1-year equity swap with quarterly payments 300 days ago. The bank agreed to pay an annual fixed rate of 4% and receive the return on an international equity index. The index was trading at 3,000 at the end of the third quarter, 30 days ago. The current 60-day MRR rate is 3.6%, the discount factor is 0.9940, and the index is now at 3,150. The value of the swap to the bank is *closest* to:
   A. –$257,795.
   B. –$114,676.
   C. $230,300.

KEY CONCEPTS

LOS 30.a

The calculation of the forward price for an equity forward contract is different because the periodic dividend payments affect the no-arbitrage price calculation. The forward price is reduced by the future value of the expected dividend payments; alternatively, the spot price is reduced by the present value of the dividends.

FP(on an equity security) =

$$(S_0 - \text{PVD}) \times (1 + R_f)^T = \left[S_0 \times (1 + R_f)^T\right] - \text{FVD}$$

The value of an equity forward contract to the long is the spot equity price minus the present value of the forward price minus the present value of any dividends expected over the term of the contract:

$$V_t(\text{long position}) = [S_t - \text{PVD}_t] - \left[\frac{\text{FP}}{(1 + R_f)^{(T-t)}}\right]$$

If given the current forward price (FP_t) on the same underlying and with the same maturity:

$$V_t(\text{long position}) = \left[\frac{FP_t - FP}{(1 + R_f)^t}\right]$$

We typically use the continuous time versions to calculate the price and value of a forward contract on an equity index using a continuously compounded dividend yield.

$$FP(\text{on an equity index}) = S_0 \times e^{(R_f^c - \delta^c) \times T} = (S_0 \times e^{-\delta^c \times T}) \times e^{R_f^c \times T}$$

LOS 30.b

Forward price = spot price + net cost of carry

For a security without underlying cash flows:

$FP = S_0 \, (1+R_f)^T$

For a security with underlying cash flows:

$FP = (S_0 - PVC) \times (1+R_f)^T$

where PVC = present value of the cash flow on the security.

LOS 30.c

The "price" of an FRA is the implied forward rate for the period beginning when the FRA expires to the maturity of the underlying "loan."

The value of an FRA at maturity is the interest savings to be realized at maturity of the underlying "loan" discounted back to the date of the expiration of the FRA at the current MRR. The value of an FRA before maturity is the interest savings estimated by the implied forward rate discounted back to the valuation date at the current MRR.

LOS 30.d

For forwards on coupon-paying bonds, the price is calculated as the spot price minus the present value of the coupons times the quantity one plus the risk-free rate:

$$FP(\text{on a fixed income security}) = (S_0 - PVC) \times (1 + R_f)^T = S_0 \times (1 + R_f)^T - FVC$$

The value of a forward on a coupon-paying bond t years after inception is the spot bond price minus the present value of the forward price minus the present value of any coupon payments expected over the term of the contract:

$$V_t(\text{long position}) = [S_t - PVC_t] - \left[\frac{FP}{(1 + R_f)^{(T-t)}}\right]$$

If given the current forward price (FP_t) on the same underlying and with the same maturity:

$$V_t(\text{long position}) = \left[\frac{FP_t - FP}{(1 + R_f)^t}\right]$$

In a futures contract, the short may have delivery options (to decide which bond to deliver). In such a case, the quoted futures price is adjusted using the conversion factor for the cheapest-to-deliver bond:

$$QFP = FP/CF = [(\text{full price})(1 + R_f)^T - AI_T - FVC]\left(\frac{1}{CF}\right)$$

where:
the full price = clean price + accrued interest at t = 0
AI_T = accrued interest at futures contract maturity.

LOS 30.e

The fixed periodic-rate on an *n*-period swap at initiation (as a percentage of the principal value) can be calculated as:

$$SFR(\text{periodic}) = \frac{1 - \text{final discount factor}}{\text{sum of discount factors}}$$

where:

$$\text{discount factor} = Z = \frac{1}{\left[1 + \left(MRR \times \frac{\text{days}}{360}\right)\right]}$$

The value of a swap *on* a payment date has a simple relationship to the difference between the new swap fixed rate and the original swap fixed rate:

$$\text{value to the payer} = \Sigma Z \times (SFR_{New} - SFR_{Old}) \times \frac{\text{days}}{360} \times \text{notional principal}$$

where:
ΣZ = sum of discount factors associated with the *remaining* settlement periods

LOS 30.f

The fixed rates in a fixed-for-fixed currency swap are determined using the yield curves for the relevant currencies. The notional principal amounts in the two currencies are of equal value, based on the exchange rate at inception of the swap. Use the difference in values to value the currency swap. The conversion of value from one of the two currencies into the common currency is based on the exchange rate on the valuation date.

LOS 30.g

The fixed-rate side of an equity swap is priced and valued just like an interest rate swap. The equity side can be valued by multiplying the notional amount of the contract by 1 + the percentage equity appreciation since the last payment date. Use the difference in values to value the swap.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 30.1, 30.2

1. **C** The dividend in 65 days occurs after the contract has matured, so it's not relevant to computing the forward price.

$$PVD = \frac{\$0.30}{1.05^{20/365}} = \$0.2992$$

$$FP = (\$30.00 - \$0.2992) \times 1.05^{60/365} = \$29.94$$

(Module 30.2, LOS 30.a)

2. **B**

$$V(\text{long position}) = \$21.00 - \left[\frac{\$29.94}{1.05^{23/365}}\right] = -\$8.85$$

$$V(\text{short position}) = +\$8.85$$

(Module 30.2, LOS 30.a)

3. **B** Use the dividend rate as a continuously compounded rate to get:

$$FP = 540 \times e^{(0.07 - 0.018) \times (200 / 365)} = 555.61$$

(Module 30.2, LOS 30.a)

4. **B** The value of the long position in a forward contract on a stock at time *t* is:

$$V_t(\text{long position}) = [S_t - PVD_t] - \left[\frac{FP}{(1 + R_f)^{(T-t)}}\right]$$

If the dividends are ignored, the *long* position will be overvalued by the present value of the dividends; that means the *short* position (which is what the question asks for) will be undervalued by the same amount. (Module 30.2, LOS 30.a)

5. **A** The dividend in 125 days is irrelevant because it occurs after the forward contract matures.

$$PVD = \frac{\$0.50}{1.04^{35/365}} = \$0.4981$$

$$FP = (\$35 - \$0.4981) \times 1.04^{120/365} = \$34.95$$

$$V_{45}(\text{short position}) = -\left(\$27.50 - \frac{\$34.95}{1.04^{75/365}}\right) = \$7.17$$

(Module 30.2, LOS 30.a)

Module Quiz 30.3

1. **B** Remember that U.S. Treasury bonds make semiannual coupon payments, so:

$$C = \frac{\$1{,}000 \times 0.06}{2} = \$30.00$$

$$PVC = \frac{\$30.00}{1.05^{98/365}} = \$29.61$$

The forward price of the contract is therefore:

FP (on a fixed income security)

$= (S_0 - PVC) \times (1 + R_f)^T = (\$1{,}044 - \$29.61) \times (1.05)^{120/365}$

$= \$1{,}030.79$

(LOS 30.d)

Module Quiz 30.4

1. **C** The actual (unannualized) rate on the 90-day loan is:

$$R90 = 0.05 \times \frac{90}{360} = 0.0125$$

The actual rate on the 150-day loan is:

$$R150 = 0.06 \times \frac{150}{360} = 0.025$$

The price of the 3 × 5 FRA (the 60-day forward rate in 90 days) is:

$$\left(\frac{1.025}{1.0125} - 1\right) \times \frac{360}{60} = 0.074 = 7.4\%$$

(LOS 30.c)

2. **C** A 3 × 6 FRA expires in 90 days and is based on 90-day MRR, so it is the appropriate hedge for 90-day MRR 90 days from today. The rate is calculated as:

$$R90 = 0.047 \times \frac{90}{360} = 0.0118$$

$$R180 = 0.049 \times \frac{180}{360} = 0.0245$$

$$\text{price of } 3 \times 6 \text{ FRA} = \left(\frac{1.0245}{1.0118} - 1\right) \times \frac{360}{90} = 0.0502 = 5.02\%$$

(LOS 30.c)

Module Quiz 30.5

1. **A** The original FRA will now be a 2×5 FRA (as 1 month has passed).

$$\begin{aligned}
\text{new FRA} &= \{[(1 + L150 \times 150/360) / (1 + L60 \times 60/360)] - 1\} \times [360 / 90] \\
&= \{[(1 + 0.032 \times 150/360) / (1 + 0.026 \times 60/360] - 1\} \times 4 \\
&= [(1.01333 / 1.00433) - 1] \times 4 \\
&= 3.584\% \\
\text{value (long)} &= PV[(0.03584 - 0.03) \times (90/360) \times 10m] \\
&= PV(14{,}600) \\
&= (14{,}600) / (1 + 0.032 \times 150/360) \\
&= 14{,}600 / 1.01333 = 14{,}407
\end{aligned}$$

(LOS 30.c)

Module Quiz 30.6

1. **C** The quarterly fixed rate on the swap is:

$$\frac{1 - 0.95057}{3.88060} = 0.0127 = 1.27\%$$

The fixed rate on the swap in annual terms is:

$$1.27\% \times \frac{360}{90} = 5.08\%$$

(LOS 30.e)

2. **A** Calculate the swap rate:

$$\text{semi-annual swap rate} = \frac{1 - 0.8772}{0.9756 + 0.9434 + 0.9112 + 0.8772} = 0.0331$$

$$\text{swap rate} = 0.0331 \times \frac{360}{180} = 0.0662 = 6.62\%$$

(LOS 30.e)

3. **A** Based on new rates, $\Sigma DF = 0.9780 + 0.9524 + 0.9174 = 2.8478$.

$$\text{new SFR} = \frac{1 - 0.9174}{2.8478} \times \frac{360}{180} = 0.0580$$

$$\text{value (payer) notional} = \frac{(0.0580 - 0.0662)}{2} \times 2.8478 = -0.01166 \text{ per } \$1$$

Because the value is negative, payer makes the payment. (LOS 30.e)

Module Quiz 30.7

1. **C** A receive-fixed C$ position will hedge the liability risk. That party would receive $1 million at swap inception (in exchange for $\frac{1{,}000{,}000}{0.7}$ = C$1,428,571) and pay it back at termination. The fixed-rate received will be calculated using the yield curve in Canada at the initiation of the swap. Because this is a fixed-for-floating currency swap, the receive-fixed position will pay a floating rate based on the U.S. yield curve. (LOS 30.f)

2. **A** coupon on $ fixed side = $1,000,000 × (0.042 / 4) = $10,500

value of the $ fixed side = (0.9923 × $10,500) + (0.9791 × $1,010,500)
= $999,800

coupon on € fixed side = €800,000 × (0.05 / 4) = €10,000

value of € fixed side (in €) = (0.9900 × €10,000) + (0.9736 × €810,000)
= €798,516

$$\text{value of € fixed side (in \$)} = \frac{€798{,}516}{0.75} = \$1{,}064{,}688$$

value of swap to bank = $999,800 – $1,064,688 = –$64,888

(LOS 30.f)

Module Quiz 30.8

1. **C** The quarterly 4% fixed-rate payment in 60 days will be in the amount of:

$5,000,000 × (4%/4) = $50,000

So the total cash flow at that time is $50,000 + $5,000,000 = $5,050,000.

$$\text{value of fixed-rate side} = 0.9940 \times \$5{,}050{,}000 = \$5{,}019{,}700$$

$$\text{value of index return side} = \$5{,}000{,}000 \times \frac{3{,}150}{3{,}000} = \$5{,}250{,}000$$

$$\text{value of swap to bank} = \$5{,}250{,}000 - \$5{,}019{,}700 = \$230{,}300$$

(LOS 30.g)

READING 31

VALUATION OF CONTINGENT CLAIMS

EXAM FOCUS

This topic review covers the valuation of options. Candidates need to be able to calculate value of an option using the binomial tree framework and should also understand the inputs into the Black-Scholes model and how they influence the value of an option. While this topic review is somewhat quantitative, candidates need to understand the material conceptually as well. This reading has a lot of testable material.

PROFESSOR'S NOTE

We cover these three LOS here together, for ease of exposition.

MODULE 31.1: THE BINOMIAL MODEL

Video covering this content is available online.

LOS 31.a: Describe and interpret the binomial option valuation model and its component terms.

LOS 31.b: Describe how the value of a European option can be analyzed as the present value of the option's expected payoff at expiration.

LOS 31.d: Calculate the no-arbitrage values of European and American options using a two-period binomial model.

BINOMIAL MODEL

A **binomial model** is based on the idea that, over the next period, the value of an asset will change to one of two possible values (binomial). To construct a binomial model, we need to know the beginning asset value, the size of the two possible changes, and the probability of each of these changes occurring.

One-Period Binomial Model

Consider a share of stock currently priced at $30. Suppose also that the size of the possible price changes, and the probability of these changes occurring are as follows:

S_0 = current stock price = $30

U = up move factor = (1 + % up) = S^+/S = 1.333

D = down move factor = (1 − % down) = S^-/S = 0.75

S^+ = stock price if an up move occurs = $S_0 \times U = 30 \times 1.333 = \$40$

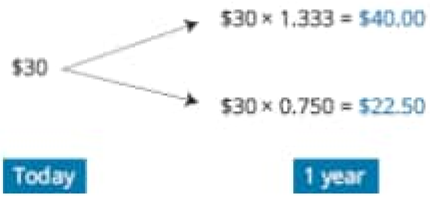
S^- = stock price if a down move occurs = $S_0 \times D = 30 \times 0.75 = \$22.50$

π_U = probability of an up move

π_D = probability of a down move = $(1 - \pi_U)$

A one-period binomial tree for this stock is shown in Figure 31.1. The beginning stock value of $30 is to the left, and to the right are the two possible paths the stock can take, based on that starting point and the size of an up- or down-move. If the stock price increases by a factor of 1.333 (a return of 33.3%), it ends up at $40.00; if it falls by a factor of 0.75 (a return of −25%), it ends up at $22.50.

Figure 31.1: One-Period Binomial Tree



The probabilities of an up-move and a down-move are calculated based on the size of the moves, as well as the risk-free rate, as:

$$\pi_U = \frac{1 + R_f - D}{U - D}$$

where:

R_f = periodically compounded annual risk-free rate

PROFESSOR'S NOTE

These up- and down-move probabilities are not the *actual* probabilities of up- or down-moves. They are the *risk-neutral* probabilities that are consistent with investor risk-neutrality. The distinction between actual probabilities and risk-neutral probabilities is not relevant for the exam.

We use the **expectations approach** to calculate the value of an option from this binomial tree. We do this by:

- Calculating the payoff of the option at maturity in both the up-move and down-move states.
- Calculating the expected value of the option in one year as the probability-weighted average of the payoffs in each state.
- Discounting the expected value back to today at the risk-free rate.

EXAMPLE: Calculating call option value with a one-period binomial tree

Calculate the value today of a one-year call option on a stock that has an exercise price of $30. Assume that the periodically compounded (as opposed to continuously compounded) risk-free rate is 7%, the current value of the stock is $30, the up-move factor is 1.333, and the down move factor is 0.75.

Answer:

First, we have to calculate the probabilities:

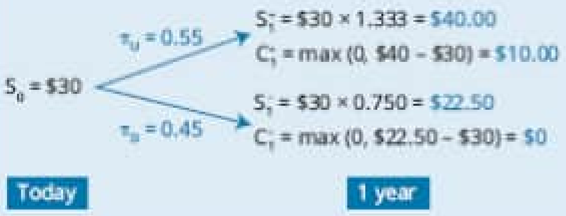
$$\pi_U = \frac{1 + R_f - D}{U - D} = \frac{1 + 0.07 - 0.75}{1.33 - 0.75} = 0.55 = 55\%$$

$$\pi_D = 1 - 0.55 = 0.45 = 45\%$$

Next, determine the option payoffs in each state. If the stock moves up to $40, the call option with an exercise price of $30 will pay off $10. If the stock moves down to $22.50, the call option will be worthless. The option payoffs are illustrated in the following figure.

Let the stock values for the up-move and down-move be S^+ and S^- and for the call values, C^+ and C^-.

With X = $30



The expected value of the option in one period is:

E(call option value in 1 year) = ($10 × 0.55) + ($0 × 0.45) = $5.50

The value of the option today, after discounting at the risk-free rate of 7%, is:

$$C = \frac{\$5.50}{1.07} = \$5.14$$

We can use the same framework to value a one-period put option on the stock.

EXAMPLE: Valuing a one-period put option on a stock

Use the information in the previous example to calculate the value of a put option on the stock that has an exercise price of $30.

Answer:

If the stock moves up to $40, a put option with an exercise price of $30 will be worthless. If the stock moves down to $22.50, the put option will be worth $7.50.

The probabilities are 0.55 and 0.45 for an up- and down-move, respectively. The expected value of the put option in one period is thus:

E(put option value in 1 year) = ($0 × 0.55) + ($7.50 × 0.45) = $3.375

The value of the option today, after discounting at the risk-free rate of 7%, is:

$$P = \frac{\$3.375}{1.07} = \$3.154$$

MODULE 31.2: TWO PERIOD BINOMIAL MODEL AND PUT-CALL PARITY

Video covering this content is available online.

Put-Call Parity

Put option value can also be computed using put-call parity, which recognizes that the value of a fiduciary call (long call, plus an investment in a zero-coupon bond with a face value equal to the strike price) is equal to the value of a protective put (long stock and long put):

$$S_0 + P_0 = C_0 + PV(X)$$

Note that both options are on the same underlying stock have the same exercise price, and the same maturity.

In our previous example, PV(X) = 30 / 1.07 = $28.04. We can verify the put call parity as:

$S_0 + P_0 = \$30 + \$3.15 = \$33.15.$

$C_0 + PV(X) = \$5.14 + 28.04 = \$33.19$ (rounding accounts for the slight difference).

Put call parity can be used to create a synthetic instrument that replicates the desired instrument.

For example, a synthetic call can be created by creating a portfolio that combines a long position in the stock, a long position in a put and a short position in a zero-coupon bond with a face value equal to the strike price (i.e., borrowing the present value of the exercise price at the risk-free rate).

$$C_0 = S_0 + P_0 - PV(X)$$

EXAMPLE: Using put-call parity

A 1-year call option on the common stock of Cross Reef, Inc., with an exercise price of $60 is trading for $8. The current stock price is $62. The risk-free rate is 4%. Calculate the price of the put option implied by put-call parity.

Answer:

According to put-call parity, to prevent arbitrage, the price of the put option must be:

$$\begin{aligned} P_0 &= C_0 - S_0 + \left[\frac{X}{(1+R_f)^T}\right] \\ &= \$8 - \$62 + \frac{\$60}{1.04} \\ &= \$3.69 \end{aligned}$$

Two-Period Binomial Model

Valuing an option using a two-period binomial model requires more steps, but uses the same method:

- Calculate the stock values at the end of two periods (there are three possible outcomes, because an up-then-down move gets you to the same place as a down-then-up move).
- Calculate the three possible option payoffs at the end of two periods.

- Calculate the expected option payoff at the end of two periods (t = 2) using the up- and down-move probabilities.
- Discount the expected option payoff (t = 2) back one period at the risk-free rate to find the option values at the end of the first period (t = 1).
- Calculate the expected option value at the end of one period (t = 1) using up- and down-move probabilities.
- Discount the expected option value at the end of one period (t = 1) back one period at the risk-free rate to find the option value today (t = 0).

Let's look at an example to illustrate the steps involved.

EXAMPLE: Valuing a call option on a stock with a two-period model

Suppose you own a stock currently priced at $50 and that a two-period European call option on the stock is available with a strike price of $45. The up-move factor is 1.25 and the down-move factor is 0.80. The risk-free rate per period is 7%. Compute the value of the call option using a two-period binomial model.

Answer:

First, compute the probability of an up-move and a down-move, and then compute the theoretical value of the stock at the end of each period:

$$\pi_U = \frac{1 + R_f - D}{U - D} = \frac{1 + 0.07 - 0.80}{1.25 - 0.80} = 0.60$$

$$\pi_D = 1 - 0.60 = 0.40$$

The two-period binomial tree for the stock is shown in the following figure.

Two-Period Binomial Tree for Stock Price

We know the value of the option at expiration in each state is equal to the stock price minus the exercise price (or zero, if that difference is negative):

$$\begin{aligned}
C^{++} &= \max(0, \$78.13 - \$45.00) &= \$33.13 \\
C^{-+} &= \max(0, \$50.00 - \$45.00) &= \$5.00 \\
C^{+-} &= \max(0, \$50.00 - \$45.00) &= \$5.00 \\
C^{--} &= \max(0, \$32.00 - \$45.00) &= \$0
\end{aligned}$$

We will approach this problem by using the single-period binomial model for each period. Using this method, we can compute the value of the call option in the up-state in period one as follows:

$$C^{+} = \frac{\text{E(call option value)}}{1+R_f} = \frac{(\pi_U \times C^{++}) + (\pi_D \times C^{+-})}{1+R_f}$$

$$= \frac{(0.6 \times \$33.13) + (0.4 \times \$5.00)}{1.07}$$

$$= \frac{\$21.88}{1.07} = \$20.45$$

The value of the call in the down-state at t = 1 is computed as:

$$C^{-} = \frac{\text{E(call option value)}}{1+R_f} = \frac{(\pi_U \times C^{-+}) + (\pi_D \times C^{--})}{1+R_f}$$

$$= \frac{(0.6 \times \$5.00) + (0.4 \times \$0.00)}{1.07}$$

$$= \frac{\$3.00}{1.07} = \$2.80$$

Now we know the value of the option in both the up-state (C^+) and the down-state (C^-) one period from now. To get the value of the option today, we simply apply our methodology one more time. Therefore, bringing (C^+) and (C^-) back one more period to the present, the value of the call option today is:

$$C = \frac{\text{E(call option value)}}{1+R_f} = \frac{(\pi_U \times C^{+}) + (\pi_D \times C^{-})}{1+R_f}$$

$$= \frac{(0.6 \times \$20.45) + (0.4 \times \$2.80)}{1.07}$$

$$= \frac{\$13.39}{1.07} = \$12.51$$

The binomial tree for the call option is shown here:

Two-Period Binomial Tree for Option Price

MODULE QUIZ 31.1, 31.2

1. Zepo, Inc., stock price is currently $80. The stock price will move up by 15% or down by 13% each period. The value of a two-period European call option with an exercise price of $62 if the risk-free rate is 4% per period is *closest* to:
   A. $19.17.
   B. $22.99.
   C. $27.11.

MODULE 31.3: AMERICAN OPTIONS

American-Style Options

Video covering this content is available online.

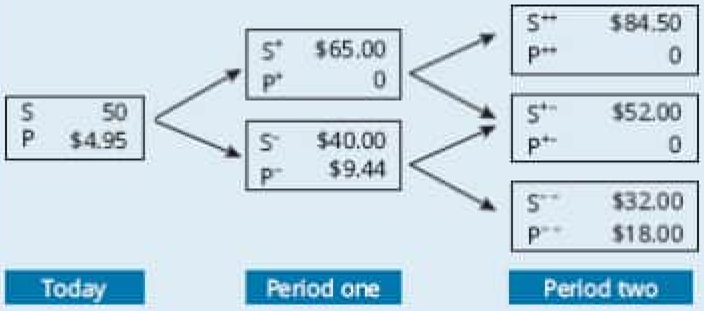
Our discussion so far has been limited to European-style options (i.e., options that can only be exercised at maturity). American-style options allow for exercise any point until maturity of the option. While the early exercise feature is not valuable for call options on non-dividend paying stocks, deep-in-the-money put options could benefit from early exercise. When an investor exercises an option early, she captures only the intrinsic value of the option and forgoes the time value. While the intrinsic value can be invested at the risk-free rate, the interest so earned is less than the time value in most cases. For a deep-in-the money put option, the upside is limited (because the stock price cannot fall below zero). In such cases, the interest on intrinsic value can exceed the option's time value.

EXAMPLE: Early exercise of a put option

Consider a stock currently trading at $50. The periodically compounded interest rate is 3%. Suppose that U = 1.3 and D = 0.80. Calculate the value of a two-period European-style put option on the stock that has an exercise price of $50. Also, determine if early exercise would make economic sense.

Answer:

The following tree shows put option value at each of the nodes in the binomial tree.



The value of the option for the down node at t = 1 is calculated as:

$$P^{-} = \frac{(\pi_U \times P^{+-} + \pi_D \times P^{--})}{(1 + R_f)} = \frac{(0.46 \times 0 + 0.54 \times 18)}{(1.03)} = \$9.44$$

The value of the option at time = 0 is calculated as:

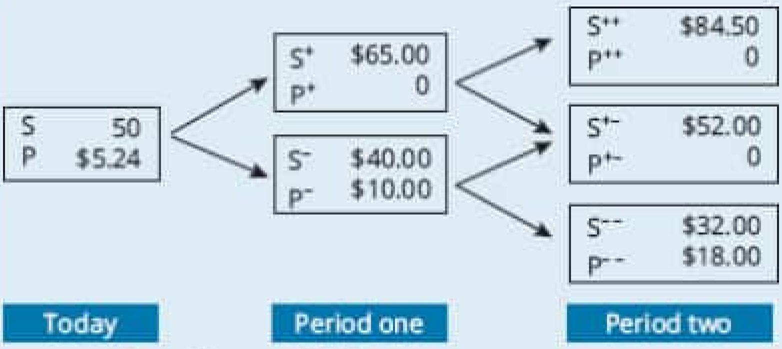
$$P = \frac{(\pi_U \times P^{+} + \pi_D \times P^{-})}{(1 + R_f)} = \frac{(0.46 \times 0 + 0.54 \times 9.44)}{(1.03)} = \$4.95$$

For the down move at t = 1, the exercise value of the put option is $10, calculated as Max(0, X-S) or Max(0, 50-40). Clearly, for this node, early exercise results in higher value ($10 as opposed to $9.44).

Had the option in the previous example been an American-style put option, the value would be $5.24 as shown in Figure 31.2.

Figure 31.2: Valuing an American-Style Put Option

The value of the put option at time 0 can be calculated as the present value of the expected value of the option at time t = 1.

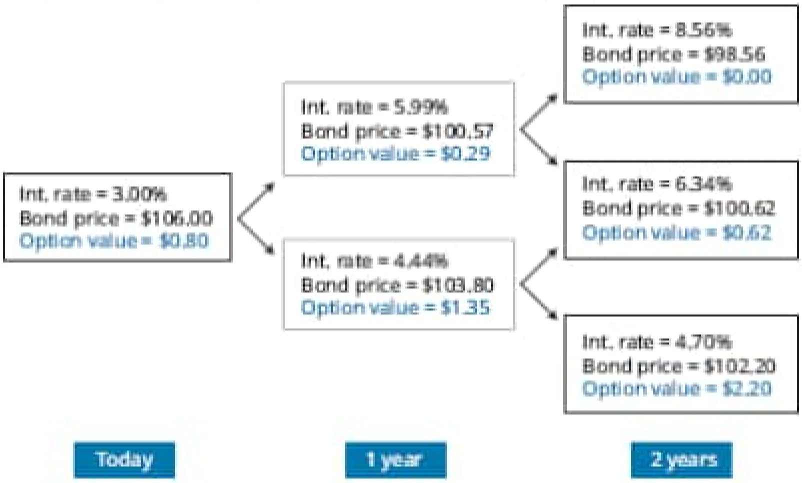


$$P_0 = \frac{(\pi_U \times P^+ + \pi_D \times P^-)}{(1 + R_f)} = \frac{(0.46 \times 0 + 0.54 \times 10.0)}{(1.03)} = \$5.24$$

American-style call options on dividend-paying stocks can be evaluated similarly: determine at each node whether the exercise value is greater than the intrinsic value and, if so, use that higher value. For dividend-paying stocks, the stock price falls when the stock goes ex-dividend, and it may make sense to exercise the call option before such a decline in price.

MODULE QUIZ 31.3

1. An analyst has calculated the value of a 2-year European call option to be $0.80. The strike price of the option is 100.00, and the underlying asset is a 7% annual coupon bond with three years to maturity. The two-period binomial tree for the European option is shown in the following figure.



The value of the comparable 2-year American call option (exercisable after 1 year) with a strike price of 100.00 is *closest* to:

A. $1.56.
B. $2.12.
C. $3.80.

MODULE 31.4: HEDGE RATIO

Video covering this content is available online.

LOS 31.c: Identify an arbitrage opportunity involving options and describe the related arbitrage.

ARBITRAGE WITH A ONE-PERIOD BINOMIAL MODEL

Let's revisit our original example of a single period binomial model. Recall that the call option has a strike price equal to the stock's current price of \$30, U = 1.333, D = 0.75, and the risk-free rate is 7%. We calculated that the probability of an up-move is 55% and that of a down-move 45%.

If the market price of the one-period \$30 call option were to be different from the \$5.14 value calculated before, there would be an arbitrage opportunity. This arbitrage will involve the call option and shares of the stock. If the option is overpriced in the market, we would sell the option and buy a fractional share of the stock for each option we sold. If the call option is underpriced in the market, we could purchase the option and short a fractional share of stock for each option purchased.

The fractional share of stock needed in the arbitrage trade (commonly referred to as the **hedge ratio** or delta), is calculated in the one-period model as:

$$h = \frac{C^+ - C^-}{S^+ - S^-}$$

EXAMPLE: Calculating arbitrage profit

In a previous example, a one-year call option on a \$30 stock (exercise price = \$30) was valued at \$5.14. We were given that the risk-free rate is 7%, and the up-move factor and down-move factor are 1.333 and 0.75, respectively. If the market price of the call option is \$6.50, illustrate how this opportunity can be exploited to earn an arbitrage profit. Assume we trade 100 call options.

Answer:

Given the up and down factors, the stock price would be \$40 and \$22.50 in the up and down states respectively in one year. The call payoffs would accordingly be \$10 and \$0 in up and down states, respectively. Because the option is overpriced, we will sell 100 call options and purchase a number of shares of stock determined by the hedge ratio:

$$h = \frac{\$10 - \$0}{\$40 - \$22.50} = 0.5714 \text{ shares per option}$$

Total number of shares to purchase = 100 × 0.5714 = 57.14

A portfolio that is long 57.14 shares of stock at \$30 per share and short 100 calls at \$6.50 per call has a net cost of:

net portfolio cost = (57.14 × \$30) − (100 × \$6.50) = \$1,064

We will borrow \$1,064 at 7%, and then will have to repay \$1,064 × (1.07) = \$1,138.48 at the end of one year. (In an arbitrage transaction, we assume that we begin with \$0.)

The values of this portfolio at maturity if the stock moves up to \$40 or down to \$22.50 are identical:

portfolio value after up-move = (57.14 × \$40) − (100 × \$10) = \$1,286

portfolio value after down-move = (57.14 × $22.50) − (100 × $0) = $1,286

Profit on the portfolio at the end of one year in either state after repayment of loan = $1,286 − $1,138.48 = $147.52.

The present value of the arbitrage profit is $147.52 / 1.07 = $137.87.

The previous calculations can be represented by the following:

$$-hS_0 + C_0 = \frac{(-hS^- + C^-)}{(1 + R_f)}$$

$$\text{Therefore, } C_0 = hS_0 + \frac{(-hS^- + C^-)}{(1 + R_f)}$$

Or equivalently,

$$C_0 = hS_0 + \frac{(-hS^+ + C^+)}{(1 + R_f)}$$

We can value the call option using the first relationship as:

$$C_0 = (0.5714 \times \$30) + \frac{(-0.5714 \times \$22.50) + \$0}{(1.07)}$$

$$= \$5.12 \textit{ (the small difference is due to rounding)}$$

Similarly, a put option can be valued as:

$$P_0 = hS_0 + \frac{(-hS^- + P^-)}{(1 + R_f)} = hS_0 + \frac{(-hS^+ + P^+)}{(1 + R_f)}$$

And the hedge ratio based on puts is:

$$h = \frac{p^+ - p^-}{s^+ - s^-}$$

MODULE QUIZ 31.4

1. In a one-period binomial model, the hedge ratio is 0.35. To construct a riskless arbitrage involving 1,000 call options if the option is "overpriced," what is the appropriate portfolio?

	Calls	Stock
A.	Buy 1,000 options	Short 350 shares
B.	Buy 1,000 options	Short 2,857 shares
C.	Sell 1,000 options	Buy 350 shares

2. A synthetic European put option is created by:
   A. buying the discount bond, buying the call option, and short-selling the stock.
   B. buying the call option, short-selling the discount bond, and short-selling the stock.
   C. short-selling the stock, buying the discount bond, and selling the call option.

MODULE 31.5: INTEREST RATE OPTIONS

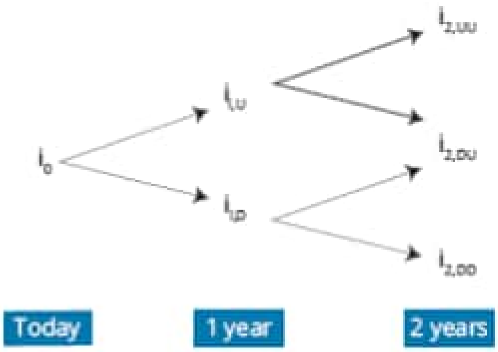
Video covering this content is available online.

LOS 31.e: Calculate and interpret the value of an interest rate option using a two-period binomial model.

WARM-UP: BINOMIAL INTEREST RATE TREES

We can use an estimate of the volatility of an interest rate to create a set of possible rate paths for interest rates in the future called a **binomial interest rate tree**. The diagram in Figure 31.3 depicts a two-period binomial interest rate tree.

Figure 31.3: Two-Period Binomial Interest Rate Tree



The interest rate at each node is a one-period forward rate. Beyond the root of the tree, there is more than one one-period forward rate for each nodal period (i.e., at Year 1, we have two 1-year forward rates, $i_{1,U}$ and $i_{1,D}$). The interest rates are selected so that the (risk-neutral) probabilities of up- and down-moves are both equal to 0.5.

PROFESSOR'S NOTE

You will not have to construct a tree for the exam—just be able to use the given tree to value an interest rate option.

Interest Rate Options

An interest rate call option has a positive payoff when the reference rate is greater than the exercise rate:

call payoff = notional principal × [Max (0, reference rate − exercise rate)]

Interest rate call options increase in value when rates increase.

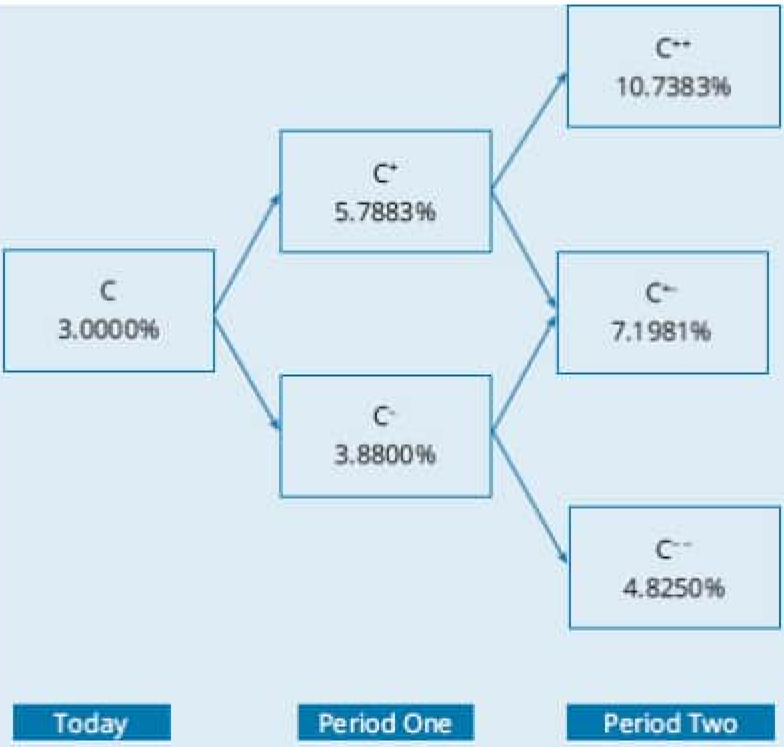
An interest rate put option has a positive payoff when the reference rate is less than the exercise rate:

put payoff = notional principal × [Max (0, exercise rate − reference rate)]

Interest rate put option values increase in value when rates decrease. Valuing interest rate options using the binomial tree is similar to valuing stock options; the value at each node is the present value of the expected value of the option. While MRR-based contracts pay interest in arrears, to keep things simple, we assume that options cash settle at maturity.

EXAMPLE: Interest rate call option valuation

Given the two-period interest rate tree shown next, what is the value of a two-period European interest rate call option with an exercise rate of 5.50% and a notional principal of $1 million? (Assume that options cash settle at time T = 2.)



Answer:

Given the exercise rate of 5.50%, the call option has a positive payoff for nodes C^{++} and C^{+-}.

The value of the option at node C^{++} can be calculated as:

[Max (0, 0.107383 – 0.055)] × $1,000,000 = $52,383

Similarly, the value at node C^{+-} can be calculated as:

[Max (0, 0.071981 – 0.055)] × $1,000,000 = $16,981

value at node C^{+} = [(0.5 × 52,383) + (0.5 × 16,981)] / (1.057883) = $32,784 (note that the discount rate is not constant)

value at node C^{-} = [(0.5 × 16,981) + 0] / (1.0388) = $8,173

value at node C = [(0.5 × 32,784) + (0.5 × 8,173)] / (1.03) = $19,882

MODULE 31.6: BLACK-SCHOLES-MERTON AND SWAPTIONS

Video covering this content is available online.

LOS 31.f: Identify assumptions of the Black–Scholes–Merton option valuation model.

The **Black-Scholes-Merton (BSM)** option valuation **model** values options in continuous time, but is based on the no-arbitrage condition we used in valuing options in discrete time with a binomial model. In the binomial model, the hedge portfolio is riskless over the next period, and the no-arbitrage option price is the one that guarantees that the hedge portfolio will yield the risk-free rate. To derive the

BSM model, an "instantaneously" riskless portfolio (one that is riskless over the next instant) is used to solve for the option price.

The assumptions underlying the Black-Scholes-Merton model are:

1. The underlying asset price follows a geometric Brownian motion process. The continuously compounded return is normally distributed. Under this framework, change in asset price is continuous: there are no abrupt jumps.
2. The (continuously compounded) risk-free rate is constant and known. Borrowing and lending are both at the risk-free rate.
3. The volatility of the returns on the underlying asset is constant and known. The price of the underlying changes smoothly (i.e., does not jump abruptly).
4. Markets are "frictionless." There are no taxes, no transactions costs, and no restrictions on short sales or the use of short-sale proceeds. Continuous trading is possible, and there are no arbitrage opportunities in the marketplace.
5. The (continuously compounded) yield on the underlying asset is constant.
6. The options are European options (i.e., they can only be exercised at expiration).

LOS 31.g: Interpret the components of the Black–Scholes–Merton model as applied to call options in terms of a leveraged position in the underlying.

The formula for valuing a European option using the BSM model is:

$$C_0 = S_0N(d_1) - e^{-rT}XN(d_2)$$

and

$$P_0 = e^{-rT}XN(-d_2) - S_0N(-d_1)$$

where

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$d_2 = d_1 - \sigma\sqrt{T}$

C_0 and P_0 = values of call and put option

T = time to option expiration

r = continuously compounded risk-free rate

S_0 = current asset price

X = exercise price

σ = annual volatility of asset returns

$N(*)$ = cumulative standard normal probability

$N(-x) = 1 - N(x)$

While the BSM model formula looks complicated, its interpretations are not:

1. The BSM value can be thought of as the present value of the expected option payoff at expiration. For a call, that means C_0 = PV $\{S_0e^{rT}N(d_1) - XN(d_2)\}$. Similarly, for a put option, P_0 = PV $\{XN(-d_2) - S_0e^{rT}N(-d_1)\}$.
2. Calls can be thought of as a leveraged stock investment where $N(d_1)$ units of stock are purchased using $e^{-rT}XN(d_2)$ of borrowed funds. (A short position in bonds can also be interpreted as borrowing funds.) A portfolio that replicates a

put option consists of a long position in $N(-d_2)$ bonds and a short position in $N(-d_1)$ stocks.

3. $N(d_2)$ is interpreted as the risk-neutral probability that a call option will expire in the money. Similarly, $N(-d_2)$ or $1 - N(d_2)$ is the risk-neutral probability that a put option will expire in the money.

EXAMPLE: BSM model

Stock of XZ, Inc., is currently trading at \$50. Suppose that the return volatility is 25% and the continuously compounded risk-free rate is 3%. Calls and puts with a strike price of \$45 and expiring in 6 months (T = 0.5) are trading at \$7.00 and \$1.00, respectively. If $N(d_1) = 0.779$ and $N(d_2) = 0.723$, calculate the value of the replicating portfolios and any arbitrage profits on both options.

Answer:

The replicating portfolio for the call can be constructed as long 0.779 shares (0.779 × \$50 = \$38.95), and borrow $45 \times e^{-0.03(0.5)} \times (0.723)$ = \$32.05.

net cost = \$38.95 – \$32.05 = \$6.90

Because the market price of the call is \$7.00, the profitable arbitrage transaction entails writing a call at \$7.00 and buying the replicating portfolio for \$6.90 to yield an arbitrage profit of \$0.10 per call.

For the put option valuation, note that $N(-d_1) = 1 - N(d_1) = 1 - 0.779 = 0.221$ and $N(-d_2) = 1 - 0.723 = 0.277$.

The replicating portfolio for the put option can be constructed as a long bond position of $45 \times e^{-0.03(0.5)} \times (0.277)$ = \$12.28 and a short position in 0.221 shares resulting in short proceeds of \$50 × 0.221 or \$11.05.

net cost = \$12.28 – \$11.05 = \$1.23

Because the market price of the put option is \$1.00, arbitrage profits can be earned by selling the replicating portfolio and buying puts, for an arbitrage profit of \$0.23 per put.

LOS 31.h: Describe how the Black–Scholes–Merton model is used to value European options on equities and currencies.

Options on Dividend Paying Stocks

So far we have assumed that the underlying stock does not pay dividends. If it does, we can adjust the model using a lowercase delta (δ) to represent the dividend yield, as follows:

$$C_0 = S_0e^{-\delta T}N(d_1) - e^{-rT}XN(d_2)$$

$$P_0 = e^{-rT}XN(-d_2) - S_0e^{-\delta T}N(-d_1)$$

where:

δ = continuously compounded dividend yield

$$d_1 = \frac{\ln(S/X) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Note that $S_0e^{-\delta T}$ is the stock price, reduced by the present value of any dividends expected to be paid during the option's life.

The put-call parity relation must also be modified if the stock pays dividends:

$$P_0 + S_0e^{-\delta T} = C_0 + e^{-rT}X$$

Options on Currencies

We can also use the Black-Scholes-Merton model to value foreign exchange options. Here, the underlying is the spot exchange rate instead of a stock price.

The value of an option on a currency can be thought of as being made up of two components, a bond component and a foreign exchange component. The value can be calculated as:

$$C_0 = S_0e^{-r(B)T}N(d_1) - e^{-r(P)T}XN(d_2)$$

and

$$P_0 = e^{-r(P)T}XN(-d_2) - S_0e^{-r(B)T}N(-d_1)$$

where:

$r(P)$ = continuously compounded price currency interest rate

$r(B)$ = continuously compounded base currency interest rate

For currencies, the carry benefit is not a dividend but rather interest earned on a deposit of the foreign currency. The spot exchange rate, S_0, is discounted at the base or foreign currency interest rate, and the bond component, $e^{-r(P)T}X$, is the exercise exchange rate discounted at the price (or domestic currency) interest rate.

LOS 31.i: Describe how the Black model is used to value European options on futures.

THE BLACK MODEL

If we ignore the mark-to-market feature of the futures contract, The Black model can be used to price European options on forwards and futures:

$$C_0 = e^{-R_f^c \times T}[F_T \times N(d_1) - X \times N(d_2)]$$

$$d_1 = \frac{\ln(F_T/X) + \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where:
σ = standard deviation of returns on the futures contract
F_T = futures price

Note that the Black model is just the BSM model with substituted $e^{-R_f^c \times T}F_T$ for S_0.

Analogous to the interpretations of the BSM model, an option on futures can be thought of as follows:

- The value of a call option on futures is equal to the value of a portfolio with a long futures position (the PV of the futures price multiplied by $N(d_1)$) and a short bond position (the PV of the exercise price multiplied by $N(d_2)$).
- The value of a put option is equal to the value of a portfolio with a long bond and a short futures position.
- The value of a call can also be thought of as the present value of the difference between the futures price (adjusted by $N(d_1)$) and the exercise price (adjusted by $N(d_2)$).

LOS 31.j: Describe how the Black model is used to value European interest rate options and European swaptions.

Interest Rate Options

Interest rate options are options on forward rates (or options on FRAs). A call option on an FRA gains when rates rise, and a put option on an FRA gains when rates fall. Interest rates are fixed in advance (i.e., at the beginning of the loan term) and settled in arrears (i.e., paid at maturity of the loan). While FRAs generally use a 30/360 convention, options on FRAs use an actual/365 convention.

Consider an interest rate call option on an (M×N) FRA expiring in M months that has a strike rate of X. The underlying is an (N−M) month forward rate. A call option on this FRA can be valued as:

$$C_0 = (AP)e^{-r(\text{actual}/365)}\left[FRA_{(M \times N)}N(d_1) - XN(d_2)\right] \times NP$$

where:
AP = accrual period = $\left[\frac{\text{actual}}{365}\right]$
NP = notional principal on the FRA

Equivalencies in Interest Rate Derivative Contracts

Combinations of interest rate options can be used to replicate other contracts, for example:

1. A long interest rate call and a short interest rate put (with exercise rate = current FRA rate) can be used to replicate a long FRA (i.e., a forward contract to receive a floating rate and pay a fixed rate).
2. Similarly, if the exercise rate = the current FRA rate, a short interest rate call and long interest rate put can be combined to replicate a short FRA position (i.e., a pay-floating, receive-fixed forward contract).
3. A series of interest rate call options with different maturities and the same exercise price can be combined to form an interest rate cap. (Each of the call options in an interest rate cap is known as a caplet.) A floating rate loan can be hedged using a long interest rate cap.
4. Similarly, an interest rate floor is a portfolio of interest rate put options, and each of these puts is known as a floorlet. Floors can be used to hedge a long position in a floating rate bond.
5. If the exercise rate on a cap and floor is same, a long cap and short floor can be used to replicate a payer swap.
6. Similarly, a short cap and long floor can replicate a receiver swap.
7. If the exercise rate on a floor and a cap are set equal to a market swap fixed rate, the value of the cap will be equal to the value of the floor.

Swaptions

A **swaption** is an option that gives the holder the right to enter into an interest rate swap. A **payer swaption** is the right to enter into a specific swap at some date in the future at a predetermined rate as the fixed-rate payer. As interest rates increase, the right to take the pay-fixed side of a swap (a payer swaption) becomes more valuable. The holder of a payer swaption would exercise it and enter into the swap if the market rate is greater than the exercise rate at expiration.

A **receiver swaption** is the right to enter into a specific swap at some date in the future as the fixed-rate receiver (i.e., the floating-rate payer) at the rate specified in the swaption. As interest rates decrease, the right to enter the receive-fixed side of a swap (a receiver swaption) becomes more valuable. The holder of a receiver swaption would exercise if market rates are less than the exercise rate at expiration.

A swaption is equivalent to an option on a series of cash flows (annuity), one for each settlement date of the underlying swap, equal to the difference between the exercise rate on the swaption and the market swap fixed rate.

If PVA represents the present value of such an annuity, the value of a payer swaption using the Black model can be calculated as:

$$\text{pay} = (\text{AP})\ \text{PVA}\ [\text{SFR}\ N(d_1) - X\ N(d_2)]\ \text{NP}$$

where:

pay = value of the payer swaption
AP = 1/# of settlement periods per year in the underlying swap
SFR = current market swap fixed rate
X = exercise rate specified in the payer swaption
NP = notional principal of the underlying swap

$$d_1 = \frac{\ln(\text{SFR}/X) + (\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

The value of a payer swaption is essentially the present value of the expected option payoff:

pay = PV (E(payoff))

Similarly, the value of a receiver swaption (which we'll call "REC") can be calculated as:

$REC = (AP)\ PVA\ [X\ N(-d_2) - SFR\ N(-d_1)]\ NP$

Equivalencies

A receiver swap can be replicated using a long receiver swaption and a short payer swaption with the same exercise rates. Conversely, a payer swap can be replicated using a long payer swaption and short receiver swaption with the same exercise rates. If the exercise rate is set such that the values of the payer and receiver swaptions are equal, then the exercise rate must be equal to the market forward swap fixed rate.

A long callable bond can be replicated using a long option-free bond plus a short receiver swaption.

MODULE QUIZ 31.5, 31.6

1. Compare the call and put prices on a stock that doesn't pay a dividend (NODIV) with comparable call and put prices on another stock (DIV) that is the same in all respects except it pays a dividend. Which of the following statements is *most accurate*? The price of a:
   A. DIV call will be less than the price of NODIV call.
   B. NODIV call will equal the price of NODIV put.
   C. NODIV put will be greater than the price of DIV put.
2. Which of the following is *not* an assumption underlying the Black-Scholes-Merton options pricing model?
   A. The underlying asset does not generate cash flows.
   B. Continuously compounded returns on the underlying are normally distributed.
   C. The option can only be exercised at maturity.

Use the following information to answer Questions 3 and 4.

Stock ABC trades for $60 and has 1-year call and put options written on it with an exercise price of $60. ABC pays no dividends. The annual standard deviation estimate is 10%, and the continuously compounded risk-free rate is 5%. The value of the call is $4.09.

Chefron, Inc., common stock trades for $60 and has a 1-year call option written on it with an exercise price of $60. The annual standard deviation estimate is 10%, the continuous dividend yield is 1.4%, and the continuously compounded risk-free rate is 5%.

3. The value of the put on ABC stock is *closest* to:
   A. $1.16.
   B. $3.28.
   C. $4.09.
4. The value of the call on Chefron stock is *closest* to:
   A. $3.51.
   B. $4.16.
   C. $5.61.

MODULE 31.7: OPTION GREEKS AND DYNAMIC HEDGING

Video covering this content is available online.

LOS 31.k: Interpret each of the option Greeks.

There are five inputs to the BSM model: asset price, exercise price, asset price volatility, time to expiration, and the risk-free rate. The relationship between each input (except the exercise price) and the option price is captured by sensitivity factors known as "the **Greeks**."

Delta describes the relationship between changes in asset prices and changes in option prices. Delta is also the hedge ratio. Call option deltas are positive because as the underlying asset price increases, call option value also increases. Conversely, the delta of a put option is negative because the put value falls as the asset price increases. See Figure 31.4.

Figure 31.4: Call and Put Delta: The Relationship Between Option Price and Underlying Asset Price

Deltas for call and put options on dividend paying stocks are given by:

$$\text{delta}_C = e^{-\delta T}N(d_1)$$
$$\text{delta}_P = -e^{-\delta T}N(-d_1)$$

where:
δ = continuously compounded dividend yield

However, because we're now using a linear relationship to estimate a non-linear change, the following relationships are only approximations:

$$\Delta C \approx e^{-\delta T}N(d_1) \times \Delta S$$
$$\Delta P \approx -e^{-\delta T}N(-d_1) \times \Delta S$$

where:
ΔC and ΔP = change in call and put price

The approximations are close for small changes in stock price, but the approximation becomes less accurate as the ΔS becomes larger.

EXAMPLE: Calculating change in option price

$e^{-\delta T}N(d_1)$ from the BSM model is 0.58. Calculate the approximate change in the price of a call option on the stock if the stock price increases by $0.75.

Answer:

$\Delta C \approx 0.58 \times \$0.75 = \$0.435$

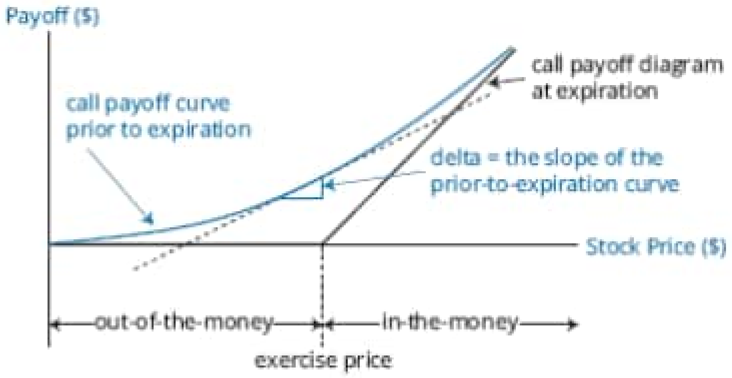
Interpreting Delta

The payoff diagrams for a European call option before- and at-expiration are shown in Figure 31.5. The "at expiration" line represents the call option's intrinsic value, which is equal to:

- Zero when the call option is out-of-the-money.
- The stock price minus the exercise price when the option is in-the-money.

Before expiration, the option also has time value, so the prior-to-expiration curve lies above the at-expiration diagram by the amount of the time value.

Figure 31.5: European Call Option Payoff Diagrams



The slope of the "prior-to-expiration" curve is the change in call price per unit change in stock price. Sound familiar? It should—that's also the definition of delta. That means *delta is the slope of the prior-to-expiration curve.* The delta of the put option is also the slope of the *prior-to-expiration* put curve.

Take a closer look at Figure 31.5. When the call option is deep out-of-the-money, the slope of the at-expiration curve is close to zero, which means the call delta will be close to zero. For deep out-of-the-money call options (i.e., when the stock price is low), the option price does not change much for a given change in the underlying stock. When the call option is in-the-money (i.e., when the stock price is high), the slope of the at-expiration curve is close to 45 degrees, which means the call delta is close to one. In this case, the call option price will change approximately one-for-one for a given change in the underlying stock price.

The bottom line is that *a call option's delta will increase from 0 to* $e^{-\delta T}$ *as stock price increases.* For a non-dividend paying stock, the delta will increase from 0 to 1 as the stock price increases.

For a put option, the put delta is close to zero when the put is out-of-the-money (i.e., when the stock price is high). When the put option is in-the-money (i.e., when the stock price is close to zero), the put delta is close to $-e^{-\delta T}$.

The bottom line is that *a put option's delta will increase from* $-e^{-\delta T}$ *to 0 as stock price increases.* For a non-dividend paying stock, the put delta increases from –1 to 0 as the stock price increases.

Now, let's consider what happens to delta as the option approaches maturity, assuming that the underlying stock price doesn't change. The effects on call and put options are different and will depend on whether the options are in- or out-of-the-money.

Remember that a call option delta is between 0 and $e^{-\delta T}$. Assuming that the underlying stock price doesn't change, if the call option is:

- **Out-of-the-money** (the stock price is less than exercise price), the call delta **moves closer to 0** as time passes.
- **In-the-money** (the stock price is greater than exercise price), the call delta **moves closer to** $e^{-\delta T}$ as time passes.

Remember that a put option delta is between $-e^{-\delta T}$ and 0. If the put option is:

- **Out-of-the-money** (the stock price is greater than exercise price), the put delta **moves closer to 0** as time passes.
- **In-the-money** (the stock price is less than exercise price), the put delta **moves closer to** $-e^{-\delta T}$ as time passes.

Gamma measures the *rate of change in delta* as the underlying stock price changes. Gamma captures the curvature of the option-value-versus-stock-price relationship. Long positions in calls and puts have positive gammas. For example, a gamma of 0.04 implies that a $1.00 increase in the price of the underlying stock will cause a call option's *delta* to increase by 0.04, making the call option more sensitive to changes in the stock price.

Gamma is highest for at-the-money options. Deep in-the-money or deep out-of-money options have low gamma. Gamma changes with stock price and with time to expiration. To lower (increase) the overall gamma of a portfolio, one should short (go long) options.

Recall that delta provides an approximation for change in option value in response to a change in the price of underlying. Including gamma in our equation would improve the precision with which change in option value would be estimated (similar to how convexity improves the estimate of change in a bond's price in response to a change in interest rate).

$$\Delta C \approx \text{call delta} \times \Delta S + \tfrac{1}{2}\,\text{gamma} \times \Delta S^2$$

$$\Delta P \approx \text{put delta} \times \Delta S + \tfrac{1}{2}\,\text{gamma} \times \Delta S^2$$

Call and put options on the same underlying asset with the same exercise price and time to expiration will have equal gammas.

Vega measures the sensitivity of the option price to changes in the volatility of returns on the underlying asset, as shown in Figure 31.6. Both call and put options are more valuable, all else equal, the higher the volatility, so vega is positive for both calls and puts. Volatility is a very important input to any option valuation model (and thus vega is an important sensitivity measure) because option values can be very sensitive to changes in volatility. Note that vega gets larger as the option gets closer to being at-the-money.

Figure 31.6: Call and Put Vega: The Relationship Between Option Price and Volatility

Rho measures the sensitivity of the option price to changes in the risk-free rate, as shown in Figure 31.7. The price of a European call or put option does not change much if we use different inputs for the risk-free rate, so rho is not a very important sensitivity measure. Call options increase in value as the risk-free rate increases (the rho of a call option is positive). You can see this by looking at the BSM model. As the risk-free rate increases, the second term decreases, and the value of the call option increases.

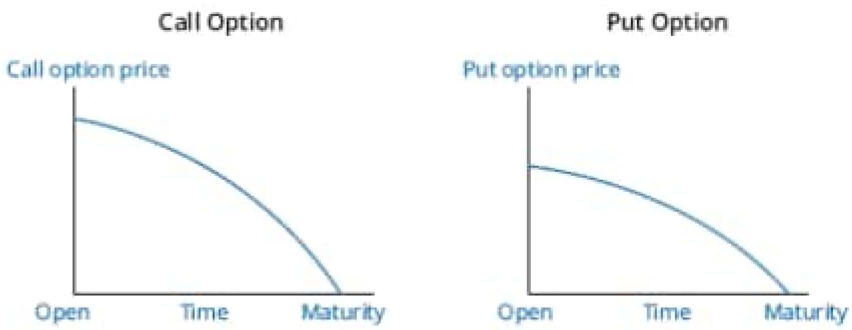
$$C_0 = \left[S_0 \times N(d_1)\right] - \left[X \times e^{-R_f^c \times T} \times N(d_2)\right]$$

Put options decrease in value as the risk-free rate increases (the rho of a put option is negative).

Figure 31.7: Call and Put Rho: The Relationship Between Option Price and the Risk-Free Rate

Theta measures the sensitivity of option price to the passage of time, as shown in Figure 31.8. As time passes and a call option approaches maturity, its speculative value declines, all else equal. This is called **time decay**. That behavior also applies for *most* put options (though *deep in-the-money* put options close to maturity may actually *increase* in value as time passes). Notice that because it is a measure of time decay, *theta is less than zero*: as time passes and the option approaches the maturity date, the option value decreases (holding other factors constant).

Figure 31.8: Call and Put Theta: The Relationship Between Option Price and the Time to Expiration



PROFESSOR'S NOTE

The statement that theta is less than zero is counterintuitive to many Level II candidates because there are two ways to state the relationship between time and option value:

- The relationship between option value and time to maturity is positive: all else equal, shorter maturity options have lower values.
- The relationship between option value and the passage of time is negative: all else equal, as time passes and the option approaches maturity, the value of the option decays.

The key to understanding why theta is less than zero is to recognize that it is capturing the intuition of the second statement.

Figure 31.9: Direction of European Option Prices for a Change in the Five Black-Scholes-Merton Model Inputs

Sensitivity Factor (Greek)	Input	Calls	Puts
Delta	Asset price (S)	Positively related Delta > 0	Negatively related Delta < 0
Gamma	Delta	Positive Gamma > 0	Positive Gamma > 0
Vega	Volatility (σ)	Positively related Vega > 0	Positively related Vega > 0
Rho	Risk-free rate (r)	Positively related Rho > 0	Negatively related Rho < 0
Theta	Time to expiration (T)	Time value → \$0 as call → maturity Theta < 0	Time value →\$0 as put → maturity Theta < 0*
	Exercise price (X)	Negatively related	Positively related

* There is an exception to the general rule that European put option thetas are negative. The put value may increase as the option approaches maturity if the option is deep in-the-money and close to maturity.

LOS 31.l: Describe how a delta hedge is executed.

Dynamic Hedging

The goal of a **delta-neutral portfolio** (or **delta-neutral hedge**) is to combine a long position in a stock with a *short* position in a call option so that the value of the portfolio does not change as the stock price changes. The number of calls to sell is equal to:

$$\text{number of short call options needed to delta hedge} = \frac{\text{number of shares hedged}}{\text{delta of call option}}$$

Delta-neutral portfolios can also be constructed using put options. In such a case, a long stock position is combined with a *long* position in puts. (Remember that put deltas are negative).

$$\text{number of long put options needed to delta hedge} = -\frac{\text{number of shares}}{\text{delta of the put option}}$$

Delta neutral portfolios are best illustrated with an example.

EXAMPLE: Hedging with call options, part 1

Suppose you own 60,000 shares of Arthurall Company common stock that is currently selling for $50. A call option on Arthurall with a strike price of $50 is selling for $4 and has a delta of 0.60. Determine the number of call options necessary to create a delta-neutral hedge.

Answer:

In order to determine the number of call options necessary to hedge against instantaneous movements in Arthurall's stock price, we calculate:

$$\text{number of options needed to delta hedge} = \frac{60{,}000}{0.60}$$

$$= 100{,}000 \text{ options, or } 1{,}000 \text{ option contracts}$$

Because we are long the stock, we need to short the options.

EXAMPLE: Hedging with call options, part 2

Calculate the effect on portfolio value of a $1 increase in the price of Arthurall stock.

Answer:

If the price of Arthurall stock increased instantly by $1.00, the value of your call option position would decrease by $0.60 because you have sold the call option contracts. Therefore, the net impact of the price change on the value of the hedged portfolio would be zero:

total change in value of stock position = 60,000 × $1 = +$60,000

total change in value of option position = 100,000 × –$0.60 = –$60,000

total change in portfolio value = $60,000 – $60,000 = $0

A key consideration in delta-neutral hedging is that the delta hedged asset position is only risk free for a very small change in the value of the underlying stock. The delta-neutral portfolio must be continually rebalanced to maintain the hedge; for this reason, it is called a dynamic hedge. As the underlying stock price changes, so does the delta of the call option, and thus too the number of calls that need to be sold to maintain a hedged position. Hence, continuously maintaining a delta-neutral position involves significant transaction costs.

LOS 31.m: Describe the role of gamma risk in options trading.

If the assumptions of the BSM hold, changes in stock price will be continuous rather than abrupt, and hence there will be no gamma risk. In this context, gamma can be viewed as a measure of how poorly a dynamic hedge will perform when it is not rebalanced in response to a change in the asset price. **Gamma risk** is therefore the risk that the stock price might abruptly "jump," leaving an otherwise delta-hedged portfolio unhedged.

Consider a delta hedge involving a long position in stock and short position in calls. If the stock price falls abruptly, the loss in the long stock position will not equal the gain in the short call position. This is the gamma risk of the hedge.

Because a stock's delta is always 1, its gamma is 0. A delta-hedged portfolio with a long position in stocks and a short position in calls will have negative net gamma exposure.

LOS 31.n: Define implied volatility and explain how it is used in options trading.

Future volatility is one of the inputs in the BSM model. Estimates based on historical volatility are often used because future volatility is not directly observable.

Implied volatility is the standard deviation of continuously compounded asset returns that is "implied" by the market price of the option. Of the five inputs into the BSM model, four are observable: stock price, exercise price, risk-free rate, and time to maturity. If we plug these four inputs into the BSM model, we can solve for the value of volatility that makes the BSM model value equal the market price.

Volatility enters the BSM pricing equation in a complex way, and there is no closed-form solution for the volatility that will produce a value equal to the market price. Rather, implied volatility must be found by iteration (i.e., trial and error).

Traders often use implied volatilities to gauge market perceptions. For example, implied volatilities in options with different exercise prices on the same underlying may reflect different implied volatilities (a violation of the BSM assumption of constant volatility).

Traders can also use implied volatility as a mechanism to quote option prices. In this way, options with different exercise prices and maturity dates can be quoted using the same unit of measurement.

EXAMPLE: Implied volatility

Calls on Blue, Inc., stock are currently trading at an implied volatility of 22%. A trader estimates that the future volatility will actually be closer to 25%. To capitalize on her beliefs, the trader should:

A. change her model.

B. buy calls on Blue stock.

C. write calls on Blue stock.

Answer:

B. Based on the trader's beliefs, call options on Blue stock are underpriced in the market. Accordingly, she should buy the calls.

MODULE QUIZ 31.7

1. Which of the following statements is *least accurate*? The value of a:
   A. call option will decrease as the risk-free rate increases.
   B. put option will decrease as the exercise price decreases.
   C. call option will decrease as the underlying stock price decreases.
2. Which of the following inputs into the Black-Scholes-Merton model is *least likely* to have opposite effects on put and call prices?
   A. Volatility.
   B. Strike price.
   C. Risk-free rate.
3. Which of the following statements is *most accurate*? Implied volatility:
   A. requires market prices.
   B. requires a series of past returns.
   C. is equal for otherwise identical options with different maturities.
4. The delta of a put is –0.43. If the price of the underlying stock increases from $40 to $44, the price of the put option:
   A. increases by approximately 4.3%.
   B. decreases by approximately 4.3%.
   C. decreases by approximately $1.72.
5. The current stock price of Heart, Inc., is $80. Call and put options with exercise prices of $50 and 15 days to maturity are currently trading. Which of these scenarios is *most likely* to occur if the stock price falls by $1?

	Call value	Put value
A.	Decrease by $0.94	Increase by $0.08
B.	Decrease by $0.76	Increase by $0.96
C.	Decrease by $0.07	Increase by $0.89

6. A put option with an exercise price of $45 is trading for $3.50. The current stock price is $45. What is the *most likely* effect on the option's delta and gamma if the stock price increases to $50?
   A. Both delta and gamma will increase.
   B. Both delta and gamma will decrease.
   C. One will increase and the other will decrease.
7. From the Black-Scholes-Merton model, $N(d_1) = 0.42$ for a 3-month call option on Panorama Electronics common stock. If the stock price falls by $1.00, the price of the call option will:
   A. decrease by less than the increase in the price of the put option.
   B. increase by more than the decrease in the price of the put option.
   C. decrease by the same amount as the increase in the price of the put option.

KEY CONCEPTS

LOS 31.a

To value an option using a two-period binomial model:

- Calculate the stock values at the end of two periods (there are three possible outcomes because an up-down move gets you to the same place as a down-up move).
- Calculate option payoffs at the end of two periods.

- Calculate expected values at the end of two periods using the up- and down-move probabilities. Discount these back one period at the risk-free rate to find the option values at the end of the first period.
- Calculate expected value at the end of period one using the up- and down-move probabilities. Discount this back one period to find the option value today.

To price an option on a bond using a binomial tree, (1) price the bond at each node using projected interest rates, (2) calculate the intrinsic value of the option at each node at maturity of the option, and (3) calculate the value of the option today.

LOS 31.b

Option values can be calculated as present value of expected payoffs on the option, discounted at the risk-free rate. The probabilities used to calculate the expected value are risk-neutral probabilities.

LOS 31.c

Synthetic call and put options can be created using a replicating portfolio. A replication portfolio for a call option consists of a leveraged position in *h* shares where *h* is the hedge ratio or delta of the option. A replication portfolio for a put option consists of a long position in a risk-free bond and a short position in *h* shares. If the value of the option exceeds the value of the replicating portfolio, an arbitrage profit can be earned by writing the option and purchasing the replicating portfolio.

LOS 31.d

The value of a European call option using the binomial option valuation model is the present value of the expected value of the option in the 2 states.

$$C_0 = \frac{(\pi_U \times C^+ + \pi_D \times C^-)}{(1 + R_f)}$$

The value of an American-style call option on a non-dividend paying stock is the same as the value of an equivalent European-style call option. American-style put options may be more valuable than equivalent European-style put options due to the ability to exercise early and earn interest on the intrinsic value.

LOS 31.e

The value of an interest rate option is computed similarly to the value of options on stocks: as the present value of the expected future payoff. Unlike binomial stock price trees, binomial interest rate trees have equal (risk-neutral) probabilities of the up and down states occurring.

LOS 31.f

The assumptions underlying the BSM model are:

- The price of the underlying asset changes smoothly (i.e., does not jump) and has a normally distributed continuously compounded return.
- The (continuous) risk-free rate is constant and known.
- The volatility of the underlying asset is constant and known.
- Markets are “frictionless.”
- The continuously compounded yield on the underlying asset is constant.

- The options are European.

LOS 31.g

Calls can be thought of as leveraged stock investment where $N(d_1)$ units of stock is purchased using $e^{-rT}XN(d_2)$ of borrowed funds. A portfolio that replicates a put option can be constructed by combining a long position in $N(-d_2)$ bonds and a short position in $N(-d_1)$ stocks.

LOS 31.h

European options on dividend-paying stock can be valued by adjusting the model to incorporate the yield on the stock: the current stock price is adjusted by subtracting the present value of dividends expected up until option expiration. Options on currencies incorporate a yield on the foreign currency based on the interest rate in that currency.

LOS 31.i

The Black model is simply the BSM model with $e^{-R_f^c \times T}F_T$ substituted for S_0.

LOS 31.j

The Black model can be used to value interest rate options by substituting the current forward rate in place of the stock price and the exercise rate in place of exercise price. The value is adjusted for accrual period (i.e., the period covered by the underlying rate).

A swaption is an option that gives the holder the right to enter into an interest rate swap. A payer (receiver) swaption is the right to enter into a swap as the fixed-rate payer (receiver). A payer (receiver) swaption gains value when interest rates increase (decrease).

LOS 31.k

Direction of BSM option value changes for an increase in the five model inputs:

Sensitivity Factor (Greek)	Input	Calls	Puts
Delta	Asset price (S)	Positively related Delta > 0	Negatively related Delta < 0
Gamma	Delta	Positively related Gamma > 0	Positively related Gamma > 0
Vega	Volatility (σ)	Positively related Vega > 0	Positively related Vega > 0
Rho	Risk-free rate (r)	Positively related Rho > 0	Negatively related Rho < 0
Theta	Time to expiration (T)	Time value → \$0 as call → maturity Theta < 0	Time value → \$0 as put → maturity Theta < 0
	Exercise price (X)	Negatively related	Positively related

Delta is the change in the price of an option for a one-unit change in the price of the underlying security. $e^{-\delta T}N(d_1)$ from the BSM model is the delta of a call option, while $-e^{-\delta T}N(-d_1)$ is the put option delta.

As stock price increases, delta for a call option increases from 0 to $e^{-\delta T}$, while delta for a put option increases from $-e^{-\delta T}$ to 0.

LOS 31.l

The goal of a delta-neutral portfolio (or delta-neutral hedge) is to combine a long position in a stock with a short position in call options (or a long position in put options) so that the portfolio value does not change when the stock value changes. Given that delta changes when stock price changes, a delta hedged portfolio needs to be continuously rebalanced. Gamma measures how much delta changes as the asset price changes and, thus, offers a measure of how poorly a fixed hedge will perform as the price of the underlying asset changes.

LOS 31.m

When the price of the underlying stock abruptly jumps, a violation of BSM, the delta of the option would change (captured by the option gamma), leaving a previously delta hedged portfolio unhedged. This is the gamma risk of a delta hedged portfolio.

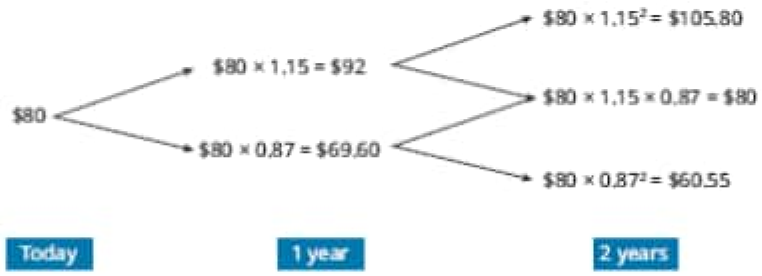
LOS 31.n

Implied volatility is the volatility that, when used in the Black-Scholes formula, produces the current market price of the option. If an option is overvalued, implied volatility is too high.

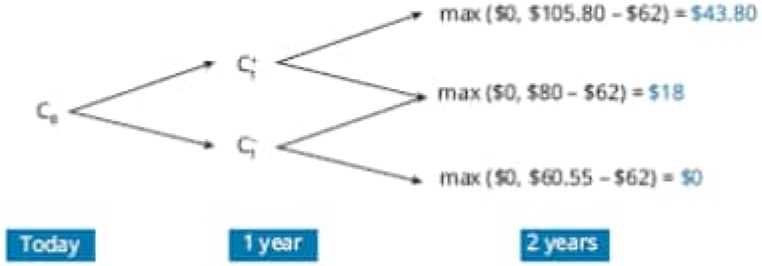
ANSWER KEY FOR MODULE QUIZZES

Module Quiz 31.1, 31.2

1. **B** **Stock Tree**



Option Tree



$$U = 1.15$$

$$D = 0.87$$

$$\pi_U = \frac{1.04 - 0.87}{1.15 - 0.87} = 0.61$$

$$\pi_D = 1 - 0.61 = 0.39$$

$$C_2^{++} = \$43.80$$

$$C_2^{+-} = C_2^{-+} = \$18.00$$

$$C_2^{--} = \$0$$

$$C_1^{+} = \frac{(0.61 \times \$43.80) + (0.39 \times \$18.00)}{1.04} = \$32.44$$

$$C_1^{-} = \frac{(0.61 \times \$18.00) + (0.39 \times \$0)}{1.04} = \$10.56$$

$$C_0 = \frac{(0.61 \times \$32.44) + (0.39 \times \$10.56)}{1.04} = \$22.99$$

(Module 31.2, LOS 31.d)

Module Quiz 31.3

1. **B** In the upper node at the end of the first year, the European option is worth \$0.29, but the American option can be exercised and a profit of \$0.57 realized (the difference between the bond price of \$100.57 and the exercise price of \$100).

   In the lower node at the end of the first year, the European option is worth \$1.35, but the American option can be exercised and a profit of \$3.80 realized (the difference between the bond price of \$103.80 and the exercise price of \$100).

   The value of the American option today is therefore:

$$\text{American option price} = \frac{(\$0.57 \times 0.5) + (\$3.80 \times 0.5)}{1.03} = \$2.12$$

(LOS 31.d)

Module Quiz 31.4

1. **C** The hedge ratio in a one-period model is equivalent to a delta, the ratio of the call price change to the stock price change. We will sell the 1,000 calls because they are overpriced. Buying 350 shares of stock will produce a riskless hedge. The payoff at expiration will return more than the riskless rate on the net cost of the hedge portfolio. Borrowing to finance the hedge portfolio and earning a higher rate than the borrowing rate produces arbitrage profits. (LOS 31.c)

2. **A** A synthetic European put option is formed by:
   - Buying a European call option.
   - Short-selling the stock.
   - Buying (i.e., investing) the present value of the exercise price worth of a pure-discount riskless bond.

   (LOS 31.c)

Module Quiz 31.5, 31.6

1. **A** The dividend affects option values because if you own the option, you do not have access to the dividend. Hence, if the firm pays a dividend during the life of the option, this must be considered in the valuation formula. Dividends decrease the value of call options, all else equal, and they increase the value of put options. (Module 31.6, LOS 31.h)

2. **A** To derive the BSM model, we need to assume no arbitrage is possible and that:
   - A continuously compounded return on the underlying has a normal distribution.
   - The (continuous) risk-free rate is constant.
   - The volatility of the underlying asset is constant.
   - Markets are "frictionless."
   - The (continuously compounded) yield on the underlying asset is constant (and not necessarily zero).
   - The options are European (i.e., they can only be exercised at maturity).

   (Module 31.6, LOS 31.f)

3. **A** According to put/call parity, the put's value is:

   $$P_0 = C_0 - S_0 + (X \times e^{-R_f^c \times T}) = \$4.09 - \$60.00 + [\$60.00 \times e^{-(0.05 \times 1.0)}] = \$1.16$$

   (Module 31.6, LOS 31.h)

4. **A** ABC and Chefron stock are identical in all respects except Chefron pays a dividend. Therefore, the call option on Chefron stock must be worth less than the call on ABC (i.e., less than $4.09). $3.51 is the only possible answer. (Module 31.6, LOS 31.h)

Module Quiz 31.7

1. **A** The value of a call and the risk-free rate are positively related, so as the risk-free rate increases, the value of the call will increase. (LOS 31.k)

2. **A** Volatility increases will increase the values of both puts and calls. (LOS 31.k)

3. **A** Implied volatility is the volatility that produces market option prices from the BSM model. Its use for pricing options is limited because it is based on market prices. Past returns are used to calculate historical volatility. (LOS 31.n)

4. **C** The put option will decrease in value as the underlying stock price increases: $-0.43 \times \$4 = -\$1.72$. (LOS 31.k)

5. **A** The call option is deep in-the-money and must have a delta close to one. The put option is deep out-of-the-money and will have a delta close to zero. Therefore, the value of the in-the-money call will decrease by close to \$1 (e.g., \$0.94), and the value of the out-of-the-money put will increase by a much smaller amount (e.g., \$0.08). The call price will fall by more than the put price will increase. (LOS 31.k)

6. **C** The put option is currently at-the-money because its exercise price is equal to the stock price of \$45. As stock price increases, the put option's delta (which is less than zero) will increase toward zero, becoming less negative. The put option's gamma, which measures the rate of change in delta as the stock price changes, is at a maximum when the option is at-the-money. Therefore, as the option moves out-of-the-money, its gamma will fall. (LOS 31.k)

7. **A** If $\Delta S = -\$1.00$, $\Delta C \approx 0.42 \times (-1.00) = -\$0.42$, and $\Delta P \approx (0.42 - 1) \times (-1.00) = \$0.58$, the call will decrease by less (\$0.42) than the increase in the price of the put (\$0.58). (LOS 31.k)

Topic Quiz: Derivatives

You have now finished the Derivatives topic section. Please log into your Schweser online dashboard and take the Topic Quiz on this section. The Topic Quiz provides immediate feedback on how effective your study has been for this material. Questions are more exam-like than typical Module Quiz or QBank questions; a score of less than 70% indicates that your study likely needs improvement. These tests are best taken timed; allow three minutes per question.

READING 32

INTRODUCTION TO COMMODITIES AND COMMODITY DERIVATIVES

EXAM FOCUS

This topic review will help you understand different commodity sectors and key factors influencing prices in those sectors. Pay special attention to what backwardation and contango mean in terms of spot and futures prices. You should understand the different components of returns to commodity futures and what determines whether roll return is positive or negative. Finally, familiarize yourself with the Insurance Theory, the Hedging Pressure Hypothesis, and the Theory of Storage and what they say about futures prices.

MODULE 32.1: INTRODUCTION AND THEORIES OF RETURN

Video covering this content is available online.

LOS 32.a: Compare characteristics of commodity sectors.

Fundamental Analysis

Supply and demand shocks affect commodity prices. Fundamental analysis seeks to predict these shocks using a variety of inputs/tools such as the following:

- *Announcements.* Government or private agencies produce forecasts, which can be useful for analysts in their models.
- *Component analysis.* Different products that make up the commodity class are the components of that commodity (e.g., jet fuel, diesel, gasoline, and lubricants are components of crude oil). Analysis of demand and supply forecasts of components can fine-tune the forecasts for the aggregate.
- *Timing issues.* Incorporating any seasonality and previously observed logistical issues can refine estimates of demand/supply.
- *Macro.* Macroeconomic factors such as inflation may trigger an increase in demand for commodities. Low interest rates may increase capital spending and, therefore, the demand for commodities in the intermediate term.

Commodities can be classified by their characteristics into sectors, including:

- Energy—crude oil, natural gas, and refined petroleum products.
- Industrial metals—aluminum, nickel, zinc, lead, tin, iron, and copper.

- Grains—wheat, corn, soybeans, and rice.
- Livestock—hogs, sheep, cattle, and poultry.
- Precious metals—gold, silver, and platinum.
- Softs (cash crops)—coffee, sugar, cocoa, and cotton.

The factors that influence supply and demand and the nature of production differ for these sectors. A summary of these differences can help explain the differences in price dynamics among the sectors.

The **energy sector** comprises crude oil, natural gas, and refined products. It is the sector with the greatest market value and is a very important source of revenue to many countries and regions.

Crude oil from different regions has different characteristics. Light oil (low viscosity) and sweet oil (low sulfur content) are less costly to refine and, therefore, sell at a premium relative to heavier or higher sulfur crude oils. Crude oil can be stored indefinitely by keeping it in the ground and is also stored in tanks and aboard tanker ships. Many countries store large amounts of crude oil as strategic reserves.

The supply of crude oil has been augmented by advances in drilling and extraction technology, especially in the 21st century. While global economic growth is an important driver of worldwide demand for oil, other factors have slowed this growth in demand. Improvements in refining technology have tended to increase the output of petroleum distillates from each barrel of crude oil, and improved engines are able to produce more work from each gallon of these distillates.

Economic cycles also affect the demand for oil, which is higher during expansions when credit is widely available and can decrease sharply when contractions lead to reductions in the availability of credit.

Improvements in the efficiency of alternative sources of energy production have also reduced the overall growth in the demand for oil. Increasingly stringent restrictions on oil exploration and production in response to environmental concerns have tended to increase the cost of oil production and decrease supply.

Political risk is an important factor in oil supply. Over half the crude oil supply comes from countries in the Middle East, and conflict there can reduce supply dramatically.

Refined products such as gasoline, heating oil, and jet fuel, are only stored for short periods. Refinery output is the relevant supply consideration. The geographic concentration of refinery capacity means that extreme weather in some coastal regions can significantly affect the supply of refined products.

Seasonal factors affect the demand for refined products in that greater vacation travel in the summer months increases gasoline demand, and colder weather in the winter increases the demand for heating oil.

Unlike crude oil, **natural gas** can be used just as it comes out of the ground with very little processing. Transportation costs play an important role in energy pricing. Crude oil can be transported at a relatively low cost on ships, while natural gas must be cooled to its liquid state to be transported by ship, significantly increasing the cost of transport.

The supply of *associated gas*—gas produced in conjunction with the extraction of crude oil—is tied to the production of crude oil. *Unassociated gas* is produced from formations where oil is not present so that its supply is not tied to the demand for and production of crude oil.

Worldwide demand and supply for gas depends on many of the same factors as supply and demand for crude oil, but seasonality due to weather is more pronounced. Cold winters increase the demand for gas for heating fuel. Hot summers increase the demand for gas as well (for cooling) because gas is a primary source of fuel for electrical power generation.

Demand for **industrial metals** is primarily tied to GDP growth and business cycles because these metals are used extensively in construction and manufacturing. Storage of metals is not costly.

Political factors, especially union strikes and restrictive environmental regulations, can have a significant effect on the supply of an industrial metal. Industrial metals must be smelted from mined ore. Both mines and smelters are large-scale operations with high development costs and high fixed costs.

Grains are grown over an annual cycle and stored, although multiple crops in a single year are possible in some areas. The risks to grain supply are the usual: droughts, hail, floods, pests, diseases, changes in climate, and so on. It would be difficult to overstate the importance of grains in feeding the world's population, especially given the potential for political instability when grain stocks are insufficient.

Precious metals are used in electronics and for jewelry and can be stored indefinitely. Gold has long been used as a store of value and has provided a hedge against the inflation risk of holding currency. Jewelry demand is high where wealth is being accumulated. Industrial demand for precious metals is sensitive to business cycles.

Livestock supply depends on the price of grain, which is the primary input in its production. When increasing grain prices increase the cost of feeding livestock, the rate of slaughter also increases, which leads to a decrease in price. Such a drawdown in population can result in subsequent increases in price over time.

Weather can affect the production of some animals. Disease is a source of significant risk to livestock producers, and some diseases have had a large impact on market prices.

Income growth in developing economies is an important source of growth in demand for livestock. Freezing allows the storage of meat products for a limited amount of time.

Softs refers to cotton, coffee, sugar, and cocoa, which are all grown in the warmer climates of the lower latitudes. Just as with grains, weather is the primary factor in determining production and price, but disease is a significant risk as well. Demand increases with increases in incomes in developing economies but is dependent on consumer tastes as well.

LOS 32.b: Compare the life cycle of commodity sectors from production through trading or consumption.

The life cycle of crude oil begins with the time it takes to drill a well and extract the crude. After being transported, crude oil is typically stored for no more than a few months. The next step is refining the crude oil into various fuels such as gasoline, heating oil, diesel oil, and jet fuel. These fuels must then be transported to the consumer.

Natural gas requires minimal processing after it is extracted. While natural gas often reaches the consumer through a pipeline, it can be cooled to liquid form and transported on specially constructed ships. Energy commodities are delivered year-round, but demand is seasonal to some extent.

The life cycle of industrial metals is straightforward: the extracted ore is smelted into the quality of metal that end users need. Industrial metals can be stored indefinitely in most cases, and the regular flow of output means that end users can meet their needs with monthly deliverable futures contracts.

A key characteristic of industrial metals production is economies of scale due to large, efficient mining and smelting operations. The large size projects required for efficient operation cost billions of dollars and take significant time to construct. Construction of new capacity or facilities when capacity utilization and earnings are high can result in the additional capacity coming online at or past the peak of the economic cycle. Mining and smelting operations are most efficient running near their capacity, so individual producers are hesitant to decrease production when prices fall because the peak of the economic cycle has passed or because facilities growth has created excess capacity in the industry.

Livestock production times vary with animal size, with chickens ready for slaughter after only weeks, hogs in about six months, and cattle after a few years. Freezing allows storage for some period after slaughter and allows international trade in livestock products such as frozen beef. Livestock production has a significant seasonal component.

Grain production is seasonal, so deliverable futures contracts are available on dates to coincide with the harvest. Because planting occurs five months or more before harvest, quantities harvested are set largely by expectations for demand when crops are planted. Grains can be stored for significant time periods after harvest. The six-month offset to harvest times in the northern and southern hemispheres brings crops to markets more frequently.

Production cycles and storage options for softs vary by product. Among softs, coffee offers an example of an agricultural commodity that is harvested somewhere around the world in almost every month. Coffee is stored in warehouses after transport by ship. Local coffee roasters then roast the beans and deliver to end users or to retail sales outlets. Coffee plants can take up to four years to produce the fruit that will become coffee beans, so there is a significant lag between investment in new capacity and increases in supply.

To hedge their price risk, coffee producers can sell in the futures market for delivery to a warehouse, and consumer companies can buy in the futures market and take delivery at the warehouse. Two different types of coffee beans are traded, Robusta and Arabica, with Arabica being the premium product.

LOS 32.c: Contrast the valuation of commodities with the valuation of equities and bonds.

Unlike stocks and bonds, commodities are physical assets, have no cash flows, and may incur storage and transportation costs.

Stocks and bonds (financial assets) can be valued by calculating the present value of their expected future cash flows (e.g., dividends, interest, etc.). Commodities produce no earnings or cash flows; however, the current (spot) price of a commodity can be viewed as the discounted value of the expected selling price at some future date. Storage costs for commodities can lead to forward prices that are higher the further the forward settlement date is in the future.

LOS 32.d: Describe types of participants in commodity futures markets.

Participants in commodity futures markets can be categorized as hedgers, traders and investors, exchanges, analysts, and regulators.

Traders and investors in the commodities market can be classified as informed investors—those who provide liquidity to the markets—and arbitrageurs. **Hedgers** are considered informed investors because they either produce or use the commodity. Hedgers reduce their risk by buying (going long) or selling (going short) futures contracts. A corn farmer can reduce the uncertainty about the price she will receive for her corn by selling corn futures. A cattle producer, however, would hedge his price risk by buying corn futures to reduce his uncertainty about the cost of feed for the cattle.

PROFESSOR'S NOTE

Hedgers are said to "do in the futures market what they must do in the future." A wheat farmer will need to sell wheat in the future (i.e., after the harvest) and can hedge price risk by selling futures contracts. A grain miller will need to buy wheat in the future and can hedge price risk by buying futures contracts.

Speculators take on commodity risk in futures markets and may act as informed investors, seeking to exploit an information or information processing advantage to profit from trading with hedgers. Speculators can also earn profits by providing liquidity to markets: buying futures when short hedgers (commodity producers) are selling and selling futures when long hedgers (commodity users) are buying.

Arbitrageurs in the commodity markets are often those in the business of buying, selling, and storing the physical commodities when the difference between spot and futures prices is too large or too small based on the actual cost of storing the commodity. When the difference is too large, an arbitrageur can buy and store the

commodity and sell it at its (too high) futures price. When the difference is too small, an arbitrageur can effectively "not store" the commodity by selling from his own inventory and going long futures, replacing the inventory at the future date.

Commodity exchanges operate in many of the world's financial centers to reflect the worldwide production and consumption of commodities as well as the globalization of financial markets in general. Investors can trade commodity futures on a smart phone or via a Bloomberg terminal.

Commodity market analysts, considered non-market participants, use market information to perform analytical work for various entities including governments, universities, economic forecasters, and commercial data analysis firms.

Various **commodity regulators** are responsible for the regulation of commodities markets around the world. In the U.S., the Commodities Futures Trading Commission (CFTC) is responsible for market regulation.

LOS 32.e: Analyze the relationship between spot prices and futures prices in markets in contango and markets in backwardation.

The difference between the spot (cash) market price and the futures price for a date in the future is referred to as the **basis** of that particular contract. The basis is calculated as the spot price minus the futures price and can be positive or negative. The difference between the futures price of a nearer maturity and the futures price of a more-distant maturity is known as the **calendar spread**.

When futures prices are higher at dates further in the future, the futures market (or futures curve) is said to be in **contango**. In a contango market, the calendar spread and basis are negative. Conversely, if futures prices are lower at dates further in the future, the market is said to be in **backwardation**, and the basis and calendar spread are positive.

When a futures market is in backwardation, long futures positions have a positive returns component (the "roll return," which we will describe later in this topic review). With a futures curve in backwardation, futures prices are lower than spot prices for the commodity. Since futures prices converge to spot prices over the term of a futures contract, there is a positive returns component from the passage of time.

When a futures market is in contango, so that futures prices are greater than spot prices, there is a negative returns component for long futures positions. As time passes, convergence of futures prices to spot prices (or longer-dated futures prices to nearer-term futures prices) results in a decrease in the value of a long futures position.

LOS 32.f: Compare theories of commodity futures returns.

Three theories of the determinants of returns on commodities, based on the shape of the futures curve, have been expounded: the Insurance Theory, the Hedging Pressure Hypothesis, and the Theory of Storage.

Economist John Maynard Keynes put forward the **Insurance Theory** of futures returns, which states that the desire of commodity producers to reduce their price risk drives commodity futures returns. Producers face uncertainty about the price they will receive for their output and reduce this uncertainty by selling futures contracts. This selling drives down futures prices. The Insurance Theory states that the futures prices will be less than current spot prices to provide a return to those buying futures from producers (i.e., speculators). In this view, the resulting positive return to the buyers of futures contracts is their return for providing insurance against price uncertainty to producers. Keynes contended that this results in backwardation "normally," and the situation was termed "normal backwardation" based on this theory.

The Insurance Theory was found to be lacking based on two empirical findings. The first finding is that for markets in backwardation, buying futures has not resulted in the extra returns the theory says buyers should receive for providing "insurance." The second finding is that many markets are not in backwardation but are in contango (future prices higher than spot prices), which would imply a negative return for providing insurance to producers.

The **Hedging Pressure Hypothesis** added the hedging behavior of commodity consumers to the Insurance Theory in an attempt to better explain observed futures returns. Just as a wheat farmer faces uncertainty about the price at which he will sell his wheat in the future, a baking company faces uncertainty about the price it will pay for flour in the future. To hedge its price risk, the baking company will go long wheat futures. The more commodity users hedge with long positions (buying futures), the more upward price pressure there is on the futures price. Under the Hedging Pressure Hypothesis, when producers' hedging behavior dominates, the market will be in backwardation, and when users' hedging behavior dominates, the market will be in contango.

Despite the intuitive appeal of the Hedging Pressure Hypothesis, it has some shortcomings. Producers typically face more concentrated price risk than consumers. Individual consumers will spend only a small portion of their income on a single commodity, and for commercial users of the commodity, the actual cost of the commodity may represent only a small portion of the total cost of the production.

Additionally, both producers and consumers may be speculators in the market, not just hedgers. Another problem with the Hedging Pressure Hypothesis is that hedging pressure is not observable, so we cannot directly test the hypothesis that relative hedging pressure is the cause of backwardation and contango.

The **Theory of Storage** is based on the idea that whether a futures market is in backwardation or contango depends on the relationship between the costs of storing the commodity for future use and the benefits of holding physical inventory of the commodity. When the costs of storage outweigh the benefits of holding physical inventory, futures are more attractive than current inventory, futures will trade at a higher price than spot, and the market will be in contango. Conversely, when the benefits of holding physical inventory outweigh the costs of storage, current possession is more attractive than future possession, spot prices are higher than futures prices, and the market will be in backwardation.

The benefits of having physical inventory available are referred to as a commodity's **convenience yield.** When physical stocks are low and there is a high probability that the commodity will be in short supply, the benefits of holding physical stock (and the convenience yield) are higher.

The Theory of Storage takes both the costs and benefits of holding a commodity into account in the following relation:

futures price = spot price + storage costs − convenience yield

Relative to spot prices, futures prices are higher when storage costs are higher, and futures prices are lower when the convenience yield is higher. Further, we can say that the shape of the futures price curve depends on available supply (i.e., current inventory of the commodity) along with expected future supply and demand.

Even with these three theories, we are left without a complete theory of commodity futures returns. "Hedging pressure" and "convenience yield" are not observable, and storage costs are not readily disclosed by participant firms.

MODULE QUIZ 32.1

1. The commodity sector that is *least affected* by weather risk is:
   A. grains.
   B. precious metals.
   C. refined energy products.
2. For which of the following commodities is the production and consumption cycle *least affected* by seasonality?
   A. Hogs.
   B. Coffee.
   C. Natural gas.
3. Which of the following factors is *most likely* to distinguish the valuation of a commodity from the valuation of an equity that pays no dividends?
   A. Holding costs.
   B. Discount rate.
   C. Timing of the future sale.
4. A commodity is *most likely* to be physically stored by a(n):
   A. exchange.
   B. speculator.
   C. arbitrageur.
5. A futures market in backwardation will exhibit:
   A. positive basis and positive calendar spreads.
   B. negative basis and positive calendar spreads.
   C. negative basis and negative calendar spreads.
6. Which theory of commodity futures returns is *least likely* to explain why futures markets can be in contango?
   A. Insurance Theory.
   B. Theory of Storage.
   C. Hedging Pressure Hypothesis.

MODULE 32.2: ANALYZING RETURNS AND INDEX CONSTRUCTION

Video covering this content is available online.

LOS 32.g: Describe, calculate, and interpret the components of total return for a fully collateralized commodity futures contract.

An investor who desires long exposure to a commodity price will typically achieve this exposure through a derivative investment in forwards or futures. Some physical commodities cannot be effectively purchased and stored long term, and for others, such as precious metals, derivative positions may be a more efficient means of gaining long exposure than purchasing the commodities outright and storing them long term.

The return on a derivatives position is not the same as the return on a commodity itself. The total return on a fully collateralized long futures position has three components: collateral return, price return, and roll return.

To take a position in futures, an investor must post collateral. When a futures portfolio is *fully collateralized*, the investor has posted cash or acceptable securities with a value equal to the notional value (price multiplied by contract size) of the futures contracts in the portfolio. If U.S. Treasury bills are deposited as collateral, the **collateral return** or **collateral yield** is simply the holding period yield on the T-bills.

The **price return** or **spot yield** on an investment in commodity futures is the change in spot prices (which can be proxied by futures prices on near-month contracts).

$$\text{price return} = (\text{current price} - \text{previous price}) / \text{previous price}$$

Since commodity derivative contracts expire, an investor who wants to maintain a position over time must close out the expiring futures position and reestablish a new position with a settlement date further in the future. This process is referred to as *rolling over* the position and leads to gains or losses which are termed the **roll return** or **roll yield**. The roll return can be positive if the futures price curve is in backwardation or negative if the futures price curve is in contango.

To hold the value of a long position constant, an investor must buy more contracts if the new longer-dated futures are trading at a lower price (market in backwardation) and buy fewer contracts if the new longer-dated futures are trading at a higher price (market in contango). In any event, the roll return on the contracts traded can be calculated as:

$$\text{roll return} = \frac{\text{price of expiring futures contract} - \text{price of new futures contract}}{\text{price of expiring futures contract}}$$

Roll return is modest over multiple periods but can be a significant portion of total return in a single period.

LOS 32.h: Contrast roll return in markets in contango and markets in backwardation.

Consider a situation where the manager of a portfolio of commodity futures contracts is rolling over July corn futures trading at 397 (cents per bushel) into

November corn futures trading at 406. The roll return is:

$$\frac{397 - 406}{397} = -2.27\%$$

With the corn futures market in contango, the roll return is negative.

Now consider a situation where the manager is rolling over July natural gas futures trading at 2.35 (dollars per million cubic feet) into August futures trading at 2.22. In this case the roll return is:

$$\frac{2.35 - 2.22}{2.35} = 5.53\%$$

Suppose we wanted a specific dollar exposure to natural gas, say $10,000. We would have originally gone long 10,000 / 2.35, or approximately 4,255 contracts. To maintain the dollar exposure upon rolling over into new contracts, we would have gone long 10,000 / 2.22, or approximately 4,504 contracts. Hence, when the contract is in backwardation, the roll return is positive and results in a larger number of long contracts upon rolling over.

If natural gas exposure is 8.5% of the manager's portfolio, we can calculate the **net roll return** for the portfolio as 0.085 × 5.53% = 0.47%.

LOS 32.i: Describe how commodity swaps are used to obtain or modify exposure to commodities.

Swaps can be used to increase or decrease exposure to commodities risk. Swaps are customized instruments created and sold by dealers, who may take on the risk of their swap exposure or hedge their exposure by entering into an offsetting swap contract (in which they have the opposite exposure to the risk factor) or by holding the physical commodity.

Swaps are created for which the payments between the two parties are based on various risk factors such as the excess returns on a commodity, the total return on the commodity, or a measure of price volatility.

In a **total return swap** the swap buyer (the long) will receive periodic payments based on the change in the futures price of a commodity plus the return on the collateral, in return for a series of fixed payments. Each period, the long will receive the total return on holding the commodity times a notional principal amount, net of the payment promised to the short. If the total return is negative, the long makes the promised fixed payment percentage *plus* the negative return percentage on the commodity over the period, times the notional amount.

For example, consider a total return swap on oil with a notional value of $10 million, in which for two years the long must pay 25 basis points monthly and will receive the total return on West Texas Intermediate (WTI) crude oil. If over the first month the price of WTI increases from 41.50 bbl to 42.10 bbl (+1.45%), the long will receive a net payment of (0.0145 – 0.0025) × $10 million = $120,000.

If over the second month the price of WTI decreases from 42.10 to 41.20 (–2.14%), the long must make a payment of (–0.0214 – 0.0025) × $10 million = $239,000 to the short.

Total return swaps are often used by institutions to gain exposure to the price risk of the underlying commodity, avoiding either holding the commodity or managing a long position in futures contracts over time.

PROFESSOR'S NOTE

Up to this point, the various swaps (interest rate, currency, equity) we have considered have comprised two periodic payment streams, with net payments based on the difference between the two. With the following instruments, the buyer may instead make a single payment at the initiation of the swap and then receive periodic payments based on the total returns, excess returns, or price volatility of a commodity—essentially "buying" exposure to the underlying risk factor.

In an **excess return swap**, a party may make a single payment at the initiation of the swap and then receive periodic payments of any percentage by which the commodity price exceeds some fixed or benchmark value, times the notional value of the swap. In months in which the commodity price does not exceed the fixed value, no payments are made.

In a **basis swap**, the variable payments are based on the difference between the prices of two commodities. Often the two commodities are one that has liquid traded futures available for hedging and the other (the one the swap buyer actually uses in production) with no liquid futures contracts available. Because the price changes of the two commodities are less than perfectly correlated, the difference between them (the basis) changes over time. By combining a hedge using the liquid futures with a basis swap, the swap buyer can hedge the price risk he faces from the input that does not have a liquid futures market.

In a **commodity volatility swap**, the underlying factor is the volatility of the commodity's price. If the volatility of the commodity's price is higher than the expected level of volatility specified in the swap, the volatility buyer receives a payment. When actual volatility is lower than the specified level, the volatility seller receives a payment. A similar swap settles based on variance in price levels of a commodity, with a swap buyer receiving a payment if the actual variance exceeds the fixed variance established at the onset of the swap. If the actual variance is lower, the variance seller receives a payment.

LOS 32.j: Describe how the construction of commodity indexes affects index returns.

There are several published commodity indexes. To be most useful, an index should be investable, in that an investor should be able to replicate the index with available liquid futures contracts.

The available commodity indexes differ in the following dimensions:

- Which commodities are included
- The weighting of the commodities in the index
- The method of rolling contracts over as they near expiration
- The method of rebalancing portfolio weights

While no index methodology will consistently outperform another index methodology, differences in methodology do result in returns differences, at least over shorter periods. Over long periods, differences between the mix and weights of constituent commodities in individual indexes will result in differences between returns, as some commodities outperform others.

Indexes may be equal weighted or weighted on some factor, such as the value of global production of an individual commodity or commodity sector. A production value weighted index will have more exposure to energy than to livestock or softs, for example.

With regard to roll methodology, a passive strategy may be to simply roll the expiring futures contracts into the near-month contract each month. A more active strategy would be to maximize roll return by selecting the further-out contracts with the greatest backwardation or smallest contango.

The frequency of rebalancing will also affect commodity index returns. Rebalancing portfolio weights will decrease returns when prices are trending but increase returns when price changes are choppy and mean-reverting. For this reason, price behavior across rebalancing periods will influence returns. If the prices of a commodity are choppy over short horizons but trending on a longer-term basis, frequent rebalancing may capture gains from mean reversion over the shorter periods but give up some of the gains from the trend of the commodity's price over the longer term.

While differences in index construction methodology will lead to differences among index returns over relatively shorter periods, no one methodology is necessarily superior over longer periods. Correlations between returns on different indexes have been relatively high, while correlations between commodity indexes and returns on stocks and bonds have been low.

MODULE QUIZ 32.2

1. Suppose that a commodity market exhibits the following futures curve on July 1, 20X1:
   - Spot price: 42.0
   - August futures price: 41.5
   - October futures price: 40.8
   - December futures price: 39.7

   An investor establishes a fully collateralized long position on July 1, 20X1, and maintains the position for one year. The futures curve on July 1, 20X2, is identical to the futures curve on July 1, 20X1, and calendar spreads did not change significantly during the year. The investor's total return on the position is *most likely*:

   A. equal to the collateral return.
   B. less than the collateral return.
   C. greater than the collateral return.

2. An investor enters into a swap contract under which the net payment will vary directly with the price of a commodity. This contract is *most accurately* described as a(n):

   A. basis swap.
   B. total return swap.
   C. excess return swap.

KEY CONCEPTS

LOS 32.a

Commodity sectors include energy (crude oil, natural gas, and refined petroleum products); industrial metals (aluminum, nickel, zinc, lead, tin, iron, and copper); grains (wheat, corn, soybeans, and rice); livestock (hogs, sheep, cattle, and poultry); precious metals (gold, silver, and platinum); and softs or cash crops (coffee, sugar, cocoa, and cotton).

Crude oil must be refined into usable products but may be shipped and stored in its natural form. Natural gas may be used in its natural form but must be liquefied to be shipped overseas.

Industrial and precious metals have demand that is sensitive to business cycles and typically can be stored for long periods.

Production of grains and softs is sensitive to weather. Livestock supply is sensitive to the price of feed grains.

LOS 32.b

The life cycle of commodity sectors includes the time it takes to produce, transport, store, and process the commodities.

- Crude oil production involves drilling a well and extracting and transporting the oil. Oil is typically stored for only a short period before being refined into products that will be transported to consumers.
- Natural gas requires little processing and may be transported to consumers by pipeline.
- Metals are produced by mining and smelting ore, which requires producers to construct large-scale fixed plants and purchase equipment. Most metals can be stored long term.
- Livestock production cycles vary with the size of the animal. Meat can be frozen for shipment and storage.
- Grain production is seasonal, but grains can be stored after harvest. Growing seasons are opposite in the northern and southern hemispheres.
- Softs are produced in warm climates and have production cycles and storage needs that vary by product.

LOS 32.c

In contrast to equities and bonds, which are valued by estimating the present value of their future cash flows, commodities do not produce periodic cash inflows. While the spot price of a commodity may be viewed as the estimated present value of its future selling price, storage costs (i.e., cash outflows) may result in forward prices that are higher than spot prices.

LOS 32.d

Participants in commodity futures markets include hedgers, speculators, arbitrageurs, exchanges, analysts, and regulators.

Informed investors are those who have information about the commodity they trade. Hedgers are informed investors because they produce or use the commodity. Some speculators act as informed investors and attempt to profit from having better information or a better ability to process information. Other speculators profit from providing liquidity to the futures markets.

LOS 32.e

Basis is the difference between the spot price and a futures price for a commodity. Calendar spread is the difference between futures prices for contracts with different expiration dates.

A market is in contango if futures prices are greater than spot prices, or in backwardation if futures prices are less than spot prices. Calendar spreads and basis are negative in contango and positive in backwardation.

LOS 32.f

Insurance Theory states that futures returns compensate contract buyers for providing protection against price risk to futures contract sellers (i.e., the producers). This theory implies that backwardation is a normal condition.

The Hedging Pressure Hypothesis expands on Insurance Theory by including long hedgers as well as short hedgers. This theory suggests futures markets will be in backwardation when short hedgers dominate and in contango when long hedgers dominate.

The Theory of Storage states that spot and futures prices are related through storage costs and convenience yield.

LOS 32.g

The total return on a fully collateralized long futures position consists of collateral return, price return, and roll return. Collateral return is the yield on securities the investor deposits as collateral for the futures position. Price return or spot yield is produced by a change in spot prices. Roll return results from closing out expiring contracts and reestablishing the position in longer-dated contracts.

LOS 32.h

Roll return is positive when a futures market is in backwardation because a long position holder will be buying longer-dated contracts that are priced lower than the expiring contracts. Roll return is negative when a futures market is in contango because the longer-dated contracts are priced higher than the expiring contracts.

LOS 32.i

Investors can use swaps to increase or decrease exposure to commodities. In a total return swap, the variable payments are based on the change in price of a commodity. In an excess return swap, the variable payments are based on the difference between a commodity price and a benchmark value. In a basis swap, the variable payments are based on the difference in prices of two commodities. In a commodity volatility swap, the variable payments are based on the volatility of a commodity price.

LOS 32.j

Returns on a commodity index are affected by how the index is constructed. The index components and weighting method affect which commodities have the greatest influence on the index return. The methodology for rolling over expiring contracts may be passive or active. Frequent rebalancing of portfolio weights may decrease index returns in trending markets or increase index returns in choppy or mean-reverting markets.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 32.1

1. **B** Precious metals mining and smelting are less susceptible to changing weather. Weather is an important factor in grain production with both droughts and flooding affecting crop yields. Oil refineries are concentrated in coastal areas where hurricanes and other extreme weather cause periodic refinery shutdowns. (LOS 32.a)

2. **B** Coffee has a long production cycle but is grown at warm latitudes and harvested throughout the year. Livestock production is strongly influenced by seasonality. Natural gas demand has a seasonal component due to its uses for heating and electricity generation for cooling. (LOS 32.b)

3. **A** While a commodity or a non-dividend-paying equity security can be valued in terms of the present value of its future sales price, a commodity may have holding costs, such as storage, that can result in a forward price that is higher than the spot price. (LOS 32.c)

4. **C** Arbitrageurs may store a physical inventory of a commodity to exploit differences between spot and futures prices relative to the costs of storing the commodity. (LOS 32.d)

5. **A** In backwardation, longer-dated futures contracts are priced lower than shorter-dated contracts or spot prices, resulting in positive basis and calendar spreads. (LOS 32.e)

6. **A** According to Insurance Theory, backwardation is normal because futures contract buyers should earn a positive return for protecting commodity producers (short hedgers) from price risk. The Hedging Pressure Hypothesis and the Theory of Storage can explain either backwardation or contango. (LOS 32.f)

Module Quiz 32.2

1. **C** The price return is zero because the spot price is unchanged over the life of the position. The roll return is positive because the market is in backwardation. Therefore the total return (price return + roll return + collateral return) is greater than the collateral return. (LOS 32.g)

2. **B** In a total return swap, the variable payment is based on the price of a commodity. In an excess return swap, the variable payment is based on the amount by which a commodity price is greater than a benchmark, and the payment is zero if the price is less than the benchmark. The variable payment

of a basis swap depends on the difference between two commodity prices. (LOS 32.i)

READING 33

REAL ESTATE INVESTMENTS

EXAM FOCUS

For the exam, understand the three approaches to valuation of real estate property. Familiarize yourself with the process and inputs for the direct capitalization method. Be able to describe the different types of publicly traded real estate securities and understand the advantages and disadvantages of investing in real estate through publicly traded securities. Be able to explain the types of REITs, as well as their economic value determinants, investment characteristics, principal risks, and due diligence considerations. Know the various approaches to REIT valuation and be able to calculate the value of a REIT share.

MODULE 33.1: OVERVIEW OF TYPES OF REAL ESTATE INVESTMENT

Video covering this content is available online.

LOS 33.a: Compare the characteristics, classifications, principal risks, and basic forms of public and private real estate investments.

Real Estate Types

There are two basic types of real estate: residential, and nonresidential (mostly commercial). Residential properties include single-family homes, apartments/condominiums, and manufactured housing. Nonresidential properties include office, shopping centers, factories, warehouses, agricultural and other specialty real estate.

Real estate investment can also be described in terms of a two-dimensional quadrant. In the first dimension, the investment is differentiated as either **public real estatepublic real estate** or **private real estate**. In the private market, ownership usually involves a direct investment such as purchasing property or lending money to a purchaser. Direct investments can be solely owned or indirectly owned through partnerships where the GP provides property management services and LPs are investors (e.g., pension plans). The public market does not involve direct investment; rather, ownership involves securities that serve as claims on the underlying assets. Public real estate investment includes ownership of a **real estate investment trust (REIT)**, a **real estate operating company (REOC)**, and **mortgage-backed securities (MBS)**.

The second dimension describes whether an investment involves debt or equity. An equity investor has an ownership interest in real estate or securities of an entity

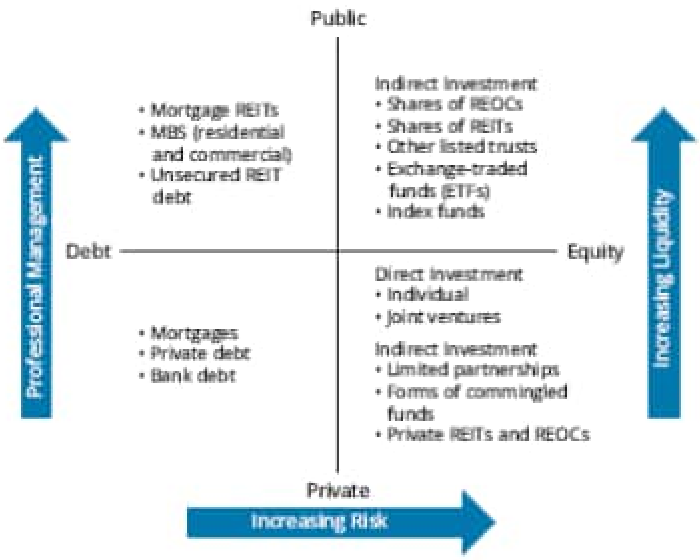
that owns real estate. Equity investors control decisions like borrowing money, property management, and the exit strategy. A debt investor is a lender that owns a mortgage or mortgage securities. Usually, the mortgage is collateralized (secured) by the underlying real estate. In this case, the lender has a superior claim over an equity investor in the event of default. Because the lender must be repaid first, the value of an equity investor's interest is equal to the value of the property less the outstanding debt.

Each of the basic forms has its own risk, expected returns, regulations, legal issues, and market structure. Private real estate investments are usually larger than public investments because real estate is indivisible and illiquid. Public real estate investments allow the property to remain undivided while allowing investors divided ownership. As a result, public real estate investments are liquid and enable investors to diversify by participating in more properties.

Private real estate investment requires property management expertise on the part of the owner or a property management company. In the case of a REIT or REOC, the real estate is professionally managed; thus, investors need no property management expertise. We will discuss REITs and REOCs later in this topic review.

Figure 33.1 summarizes the basic forms of real estate investment and can be used to identify the investment that best meets an investor's objectives.

Figure 33.1: Basic Forms of Real Estate Investment



Source: Reproduced from Level II CFA Curriculum learning module "Overview of Types of Real Estate Investment," Exhibit 2, with permission from CFA Institute.

Real Estate Characteristics

Real estate investment differs from other asset classes, such as stocks and bonds, and can complicate measurement and performance assessment.

- *Heterogeneity*. Bonds from a particular issue are alike, as are stocks of a specific company. However, no two properties are exactly the same because of location, size, age, construction materials, tenants, and lease terms.
- *High unit value*. Because real estate is indivisible, the unit value is significantly higher than stocks and bonds, which makes it difficult to construct a diversified portfolio.

- *Active management*. Investors in stocks and bonds are not necessarily involved in the day-to-day management of the companies. Private real estate investment requires active property management by the owner or a property management company. Property management involves maintenance, negotiating leases, and collection of rents. In either case, property management costs must be considered.
- *High transaction costs*. Buying and selling real estate is costly because it involves appraisers, lawyers, brokers, and construction personnel.
- *Depreciation and desirability*. Buildings wear out over time. Also, buildings may become less desirable because of location, design, or obsolescence. The deductibility of depreciation for tax purposes is an attractive feature for investors in most jurisdictions.
- *Cost and availability of debt capital*. Because of the high costs to acquire and develop real estate, property values are impacted by the level of interest rates and availability of debt capital. Real estate values are usually lower when interest rates are high and debt capital is scarce.
- *Lack of liquidity*. Real estate is illiquid. It takes time to market and complete the sale of property.
- *Difficulty in determining price*. Stocks and bonds of public firms usually trade in active markets. However, because of heterogeneity and low transaction volume, appraisals are usually necessary to assess real estate values. Even then, appraised values are often based on similar, not identical, properties. The combination of limited market participants and lack of knowledge of the local markets makes it difficult for an outsider to value property. As a result, the market is less efficient. However, investors with superior information and skill may have an advantage in exploiting the market inefficiencies.

The market for REITs has expanded to overcome many of the problems involved with direct investment. Shares of a REIT are actively traded and are more likely to reflect market value. In addition, investing in a REIT can provide exposure to a diversified real estate portfolio. Finally, investors don't need property management expertise because the REIT manages the properties.

Principal Risks

Here we'll think about the different risk factors that come with a commercial real estate investment. The purpose of identifying risks is to plan for and (ideally) mitigate them.

1. Demand and Supply Risks

- *Business conditions*. Numerous economic factors—such as gross domestic product (GDP), employment, household income, interest rates, and inflation—affect the rental market.
- *Demographics*. Shifts in size and age distribution affect the type of property in demand. Changes in socioeconomic groups and rate of new household formation all affect demand.
- *Excess supply*. Market conditions can change significantly while approvals are obtained, while the property is completed, and when the property is fully leased.

During the lead time, if market conditions weaken, the resultant lower demand affects rents and vacancy rates, resulting in lower returns. The real estate cycle is 17–18 years on average, with demand for commercial property coinciding with business cycle and demand peaking at the peak of the business cycle—resulting in high rents. The downturns, on the other hand, lead to reduction in rental cash flows.

2. Risks Relating to Valuation

- *Cost and availability of capital.* Real estate must compete with other investments for capital. As previously discussed, demand for real estate is reduced when debt capital is scarce and interest rates are high. Conversely, demand is higher when debt capital is easily obtained and interest rates are low. Thus, real estate prices can be affected by capital market forces without changes in demand from tenants.
- *Availability of information.* A lack of information when performing property analysis increases risk. The availability of data depends on the country, but generally more information is available as real estate investments become more global.
- *Lack of liquidity.* Because of the size and complexity of most real estate transactions, buyers and lenders usually perform due diligence, which takes time and is costly. A quick sale will typically require a significant discount.
- *Interest rates.* Real estate values may initially decline when interest rates rise, though that effect may reverse over time.

3. Operational Risks

- *Management expertise.* Property managers and asset managers must make important operational decisions—such as lease negotiation, property maintenance, marketing, and property renovation—when necessary.
- *Lease terms.* Some leases provide inflation protection by allowing owners to increase rent or pass through expenses because of inflation. Real estate values may not keep up with inflation when markets are weak and vacancy rates are high.
- *Leverage.* The use of debt (leverage) to finance a real estate purchase is measured by the **loan-to-value (LTV) ratio**. Higher LTV results in higher leverage and, thus, higher risk because lenders have a superior claim in the event of default. With leverage, a small decrease in net operating income (NOI) negatively magnifies the amount of cash flow available to equity investors after debt service.
- *ESG considerations.* Real estate values can be significantly reduced when a prior owner or adjacent property owner has contaminated a property.
- *Obsolescence.* Changes in technology may result in a warehouse being obsolete. An increase in cloud computing has reduced the space for local computing. Increased work-from-home during the pandemic permanently reduced the demand for office space.
- *Market disruptions.* Real estate markets change continuously as new innovations such as remote working, online shopping, and other kinds of disruptions increase and decrease the demand for various kinds of real estate.
- *Other risk factors.* Other risk factors, such as unobserved property defects, natural disasters, and acts of terrorism, may be unidentified at the time of purchase.

In some cases, insurance helps mitigate clearly identified risks. In other cases, lease provisions can shift the risk to the tenants. For example, a lease agreement could require the tenant to reimburse any unexpected operating expenses. The remaining risks need to be priced in the rental rates to earn a reasonable rate of return.

LOS 33.b: Explain portfolio roles and economic value determinants of real estate investments.

Reasons to Invest in Real Estate

- *Current income*. Investors may expect to earn income from collecting rents after paying operating expenses, financing costs, and taxes.
- *Capital appreciation*. Investors usually expect property values to increase over time, which forms part of their total return.
- *Inflation hedge*. During inflation, investors expect both rents and property values to rise.
- *Diversification*. Real estate, especially private equity investment, is less-than-perfectly correlated with the returns of stocks and bonds. Thus, adding private real estate investment to a portfolio can reduce risk relative to the expected return.
- *Tax benefits*. In some countries, real estate investors receive favorable tax treatment. For example, in the United States, the depreciable life of real estate is usually shorter than the actual life. As a result, depreciation expense is higher and taxable income is lower, resulting in lower income taxes. In addition, REITs do not pay taxes in some countries, which allows investors to escape double taxation (e.g., taxation at the corporate level and the individual level). REITs are, however, required to distribute almost all (usually >90%) of their taxable income in dividends to the shareholders.

The Role of Real Estate in a Portfolio

Real estate investment has both bond-like and stock-like characteristics. Leases are contractual agreements that usually call for periodic rental payments, similar to the coupon payments of a bond. When a lease expires, there is uncertainty regarding renewal and future rental rates. The availability of competing space, tenant profitability, and the state of the overall economy affect this uncertainty just as those same factors affect stock prices. As a result, the risk/return profile of real estate as an asset class is usually somewhere between the risk/return profiles of stocks and bonds.

A **core investing style** is a conservative real estate investment strategy that limits investments to high quality and low leverage (<30% LTV), and avoids speculative risks in favor of steady returns.

Economic Value Determinants of Real Estate Investments

National GDP growth is the largest driver of economic value for all real estate types. Overall growth in the economy means more jobs, greater need for office space, more

disposable income, more growth in shopping centers, greater demand for hotel rooms from business and leisure travelers, and so on.

In addition to economic growth, other economic factors affect the value of different types of real estate, as shown in Figure 33.2.

Figure 33.2: Major Factors Affecting Real Estate Demand by Sector

		Multifamily	Retail	Hotel	Office	Industrial
Macro factors (affect all sectors)	GDP growth	✓	✓	✓	✓	✓
	Population growth	✓	✓	✓	✓	✓
	Job creation	✓	✓	✓	✓	✓
	Wage growth	✓	✓	✓	✓	✓
	Regulatory	✓	✓	✓	✓	✓
	Taxes	✓	✓	✓	✓	✓
Individual (consumer)	Household formations	✓	✓			
	Personal income	✓	✓	✓		
	Consumer confidence	✓	✓	✓		
	Consumer credit	✓	✓	✓		
Business environment	Retail sales growth		✓			✓
	Consumer spending		✓	✓		✓
	Business formations	✓		✓	✓	✓
	Business investment			✓	✓	✓
	Business confidence			✓	✓	✓
Industrial	Industrial production					✓
	Trade, transport, and logistics					✓
	Changing supply routes					✓

Source: Reproduced from Level II CFA Curriculum learning module "Overview of Types of Real Estate Investment," Exhibit 4, with permission from CFA Institute.

While factors such as GDP growth, population growth, wage growth, taxes, and regulations affect all property types, factors such as changing supply routes and advances in logistics, trade, and transportation are unique to industrial properties.

LOS 33.c: Discuss commercial property types, including their distinctive investment characteristics.

Commercial Property Types

The basic, low-risk, commercial property types include office, industrial/warehouse, retail, and multifamily. Hospitality properties (hotels and motels) are riskier because leases are not involved and performance is highly correlated with the business cycle.

Types of commercial leases: In a gross lease, the owner is responsible for the operating expenses, and in a net lease, the tenant is responsible. In a net lease, the tenant bears the risk if the actual operating expenses are greater than expected. As a result, rent under a net lease is lower than under a gross lease. Some leases combine features from both gross and net leases. For example, the owner might pay the operating expenses in the first year of the lease. A **triple-net lease (NNN)** requires the tenants to pay their share of common area maintenance, repairs, property taxes, and building insurance. A **sale-leaseback** agreement is a long-term single-tenant lease requiring the tenant to pay all expenses directly, in addition to a base rent.

It is important to know that with all property types, location is critical in determining value. Specialty properties such as cell towers and hospitals have been excluded because of lack of substitutability of space.

- *Office*. Demand is heavily dependent on job growth, especially in industries that are heavy users of office space, such as finance and insurance. The average length of office leases varies globally.
- *Industrial*. Demand is heavily dependent on the overall economy. Import/export activity of the economy also affects demand. Net leases are common.
- *Retail*. Demand is heavily dependent on consumer spending. Consumer spending depends on the overall economy, job growth, population growth, and savings rates. Retail lease terms vary by the quality of the property, as well as the size and importance of the tenant. For example, an anchor tenant may receive favorable lease terms to attract it to the property. In turn, the anchor tenant will draw other tenants to the property.

  Retail tenants are often required to pay additional rent once sales reach a certain level. This unique feature is known as a percentage lease or percentage rent. Accordingly, the lease will specify a minimum amount of rent to be paid without regard to sales. The minimum rent also serves as the starting point for calculating the percentage rent.

  For example, suppose that a retail lease specifies minimum rent of $20 per square foot plus 5% of sales over $400 per square foot. If sales were $400 per square foot, the minimum rent and percentage rent would be equivalent ($400 sales per square foot × 5% = $20 per square foot). In this case, $400 is known as the natural breakpoint. If sales are $500 per square foot, rent per square foot is equal to $20 minimum rent + $5 percentage rent = $25. Alternatively, rent per square foot is equal to $500 sales per square foot × 5% = $25 because of the natural breakpoint.
- *Multifamily residential*. Demand depends on population growth, especially in the age demographic that typically rents apartments. The age demographic can vary by country, type of property, and locale. The cost of buying versus the cost of renting, measured by the ratio of home prices to rents, also affects the demand. As

home prices rise, there is a shift toward renting. An increase in interest rates will also make buying more expensive.

LOS 33.d: Explain the due diligence process for both private and public equity real estate investment.

Real estate investors usually perform **due diligence** to confirm the facts and conditions that might affect the value of the transaction. Due diligence may include the following:

- Review market to assess local demographics, preferences, wage growth, etc.
- Lease review and rental history.
- Confirm the operating expenses by examining bills.
- Review cash flow statements.
- Obtain an environmental report to identify the possibility of contamination.
- Perform a physical/engineering inspection to identify structural issues and check the condition of the building systems.
- Examine maintenance/service agreements to identify any recurring problem(s).
- Inspect the title and other legal documents for deficiencies.
- Have the property surveyed to confirm the boundaries and identify easements.
- Verify compliance with zoning laws, building codes, and environmental regulations.
- Verify payment of taxes, insurance, special assessments, and other expenditures.

Due diligence can be costly, but it lowers the risk of unexpected legal and physical problems.

LOS 33.e: Discuss real estate investment indexes, including their construction and potential biases.

A number of **real estate indices** are used to track the performance of the real estate asset class, including appraisal-based indices and transaction-based indices. Investors should be aware of how the indices are constructed, as well as their limitations.

Appraisal-Based Indices

Because real estate transactions covering a specific property occur infrequently, indices have been developed based on appraised values. Appraisal-based indices combine valuations of individual properties that can be used to measure market movements. A popular index in the United States is the NCREIF Property Index (NPI). Members of NCREIF, mainly investment managers and pension fund sponsors, submit appraisal data quarterly, and NCREIF calculates the return as follows:

$$\text{return} = \frac{\text{NOI} - \text{capital expenditures} + (\text{ending market value} - \text{beginning market value})}{\text{beginning market value}}$$

The return is known as a holding-period return and is equivalent to a single-period IRR. The index is then value weighted based on the returns of the separate properties.

Earlier, we found that the cap rate is equal to NOI divided by the beginning market value of the property. This is the current yield or income return of the property and is one component of the index equation. The remaining components of the equation produce the capital return. To have a positive capital return, the market value must increase by more than the capital expenditures.

The index allows investors to compare performance with other asset classes, and the quarterly returns can be used to measure risk (standard deviation). The index can also be used by investors to benchmark returns. However, a major drawback is that the income component of return does not represent REIT distributions.

Appraisal-based indices tend to lag actual transactions because actual transactions occur before appraisals are performed. Thus, a change in price may not be reflected in appraised values until the next quarter or longer if a property is not appraised every quarter. Also, appraisal lag tends to smooth the index—that is, reduce its volatility, much like a moving average reduces volatility. Finally, appraisal lag results in lower correlation with other asset classes. Appraisal lag can be adjusted by unsmoothing the index or by using a transaction-based index.

The Global Real Estate Index is a cap-weighted index, published quarterly, that includes local currency return. The index combines data from National Council of Real Estate Investment Fiduciaries (NCREIF), the European Association for Investors in Non-Listed Real Estate Vehicles (INREV), and the Asian Association for Investors in Non-Listed Real Estate Vehicles (ANREV).

Transaction-Based Indices

Transaction-based indices can be constructed using a repeat-sales index and a hedonic index.

A **repeat-sales index** relies on repeat sales of the same property. A change in market conditions can be measured once a property is sold twice. Accordingly, a regression is developed to allocate the change in value to each quarter.

A **hedonic index** requires only one sale. A regression model controls for differences in property characteristics such as size, age, location, and so forth.

Public Security Indices

Bloomberg, FTSE Russell, MSCI, Nikkei, S&P Dow Jones, and Nareit, among others, produce indices covering public securities. Indices such as CMBX also exist for real estate debt securities. Some indices cover equity REITs only, some REOCs only, or a combination of the two.

MODULE QUIZ 33.1

1. Which form of investment is *most appropriate* for a first-time real estate investor that is concerned about liquidity and diversification?
   A. Direct investment in a mixed-use development property.
   B. Ownership interest in a real estate investment trust.

C. An undivided participation interest in a commercial mortgage.

2. Ben Jacobs, CFA, is advising a client in a real estate transaction involving the purchase of a multifamily residential apartment building. Which of the following risks is *least likely* a concern for the client?
   A. Shifts in demographics.
   B. Cost of capital.
   C. Lack of diversification benefits due to higher correlation than REITs.
3. Which of the following real estate properties is *most likely* classified as commercial real estate?
   A. A residential apartment building.
   B. Cell towers.
   C. An owner-occupied, single-family home.
4. A real estate investor is concerned about rising interest rates and decides to pay cash for a property instead of financing the transaction with debt. What is the *most likely* effect of this strategy?
   A. Inflation risk is eliminated.
   B. Risk of changing interest rates is eliminated.
   C. Risk is reduced because of lower leverage.
5. Which of the following *best* describes the primary economic driver of demand for multifamily real estate?
   A. Growth in savings rates.
   B. Job growth, especially in the finance and insurance industries.
   C. Population growth.
6. Which of the following statements is *least* accurate about the NCREIF property index?
   A. The index data captures recent sales transactions.
   B. The index is value weighted.
   C. The index allows investors to compare performance with other asset classes.
7. You just entered into a contract to purchase a recently renovated apartment building, and you are concerned that some of the contractors have not been paid. In performing your due diligence, which of the following procedures should be performed to alleviate your concern?
   A. Have the property surveyed.
   B. Have an environmental study performed.
   C. Search the public records for outstanding liens.
8. Which of the following statements about real estate indices is *most accurate*?
   A. Transaction-based indices tend to lag appraisal-based indices.
   B. Appraisal-based indices tend to lag transaction-based indices.
   C. Transaction-based indices appear to have lower correlation with other asset classes compared to appraisal-based indices.
9. After overall growth in the economy, the *most* important economic factor affecting a(n):
   A. hotel REIT is job creation.
   B. storage REIT is retail sales growth.
   C. office REIT is population growth.

MODULE 33.2: INVESTMENTS IN REAL ESTATE THROUGH PUBLICLY TRADED SECURITIES

Video covering this content is available online.

Publicly Traded Real Estate Investments

Publicly traded real estate securities appeal to small investors seeking to gain exposure to professionally managed real estate. Just like mutual funds, REITs were tax advantaged under the tax codes of major developed markets and became a popular vehicle for investors to gain exposure to real estate asset class.

LOS 33.f: Discuss types of publicly traded real estate securities.

Publicly traded real estate securities can take several forms: **real estate investment trust (REIT)**, **real estate operating company (REOC)**, and residential or commercial **mortgage-backed securities (MBS)**.

We can categorize publicly traded real estate securities into two broad groups: equity and debt.

Equity

Publicly traded real estate equity securities represent ownership stakes in properties.

Equity REITs and REOCs fall into this category.

- *Equity REITs (real estate investment trusts).* REITs are tax-advantaged companies (trusts) that are for the most part exempt from corporate income tax. Equity REITs are actively managed, own income-producing real estate, and seek to profit by growing cash flows, improving existing properties, and purchasing additional properties. REITs often specialize in a particular kind of property, while still diversifying holdings by geography and other factors. To be tax-advantaged, REITs are generally required to distribute more than 90% of their taxable income to their shareholders.
- *REOCs (real estate operating companies).* REOCs are not tax advantaged; rather, they are ordinary (i.e., taxable) corporations that own real estate. A business will form as a REOC if it is ineligible to organize as a REIT. For example, the firm may intend to develop and sell real estate rather than generating cash from rental payments, or the firm may be based in a country that does not allow tax-advantaged REITs.

Debt

MBS (mortgage-backed securities) and mortgage REITs fall into this category.

- *Residential or commercial mortgage-backed securities (MBS).* Residential or commercial mortgage-backed securities are publicly traded asset-backed securitized debt obligations that receive cash flows from an underlying pool of mortgage loans. These loans may be for commercial properties (in the case of CMBS) or on residential properties (in the case of RMBS). Real estate debt securities represent a far larger aggregate market value than do publicly traded real estate equity securities.
- *Mortgage REITs.* Mortgage REITs invest primarily in mortgages, mortgage securities, or loans that are secured by real estate.

REIT Structures

Most REITs are structured as corporations or trusts and are tax-efficient conduits. In most countries, REITs are required to distribute 90% to 100% of their taxable income as dividends to be exempt from taxation. Additionally, REITs have other requirements, such as a minimum 75% investment in real estate and deriving at least 75% of their income from rental or mortgage interest. Ownership rules vary by jurisdiction but tend to disallow family-owned trusts from qualifying as REITs. In the U.S., REITs should have more than 100 owners, and no fewer than five owners can own more than 50% of the REIT shares (5/50 rule).

Advantages

Investments in REITs offer a number of advantages compared with direct investments in physical real estate:

- *Superior liquidity*. Investors in publicly traded real estate securities enjoy far greater liquidity than do investors in physical real estate, because REIT shares trade daily on a stock exchange. The low liquidity of a direct real estate investment stems from the relatively high value of an individual real estate property and the unique nature of each property.
- *Transparency*. Information about current and historical prices and volume is readily available.
- *Exemption from taxation*. As long as certain requirements are met, REITs enjoy favorable taxation, because a major part of REIT distributions is treated as a return of capital and is thus not taxable.
- *Predictable earnings*. The earnings of REITs tend to be relatively consistent over time, because REITs' rental income is fixed by contracts, unlike the income of companies in other industries.
- *Access to premium properties*. Some prestigious properties, such as high-profile shopping malls or other prominent or landmark buildings, are difficult to invest in directly. Shares in REITs that have invested in these properties represent one way to take an ownership stake in these assets.
- *Active professional management*. While a direct investment in properties requires a degree of real estate investment expertise and property management skill, REIT and REOC investments do not. REITs and REOCs employ professional management to control expenses, to maximize rents and occupancy rates, and sometimes to acquire additional properties.
- *Greater potential for diversification*. Because of the high cost of a single property, it is difficult to achieve adequate diversification through direct investments in real estate. Through REITs, however, an investor can diversify across property type and geographical location.

Disadvantages

Most of the disadvantages for REITs as an investment center around the tax code requirement that a REIT distribute most of its earnings as dividends, limiting retained earnings as a means of growth.

- *Limited potential for income growth*. REITs' high rates of income payout limit REITs' ability to generate future growth through reinvestment. This limits future income growth and may dampen the share price of REITs. A REIT may be forced

to issue equity at a disadvantageous price to finance its growth, as availability of retained earnings is limited.

- *Lack of flexibility*. The rules that qualify REITs for favorable taxation also have a downside: REITs are restricted in their investment choices. These limits may prevent REITs from being as profitable as they might otherwise be.
- *Lower diversification benefit relative to direct investment in real estate*. REITs tend to have a higher correlation with equities compared to direct real estate investments. So even though REITs allow superior diversification *amongst* property holdings, it offers poorer diversification overall *inside* an investment portfolio.

LOS 33.g: Justify the use of net asset value per share (NAVPS) in valuation of publicly traded real estate securities and estimate NAVPS based on forecasted cash net operating income.

The **net asset value per share (NAVPS)** is the (per-share) amount by which assets exceed liabilities, using current market values rather than accounting book values. NAVPS is generally considered the most appropriate measure of the fundamental value of REITs. If the market price of a REIT varies from NAVPS, the premium (or discount) reflects an investor's view of the management capabilities, leverage, and governance. REITs with high leverage and trading at a discount to NAVPS may find it difficult to refinance their maturing debt.

NAVPS is superior to book value per share (BVPS) because BVPS relies on the depreciated historical cost of a REIT's asset.

Estimating NAVPS Based on Forecasted Cash Net Operating Income

In the absence of a reliable appraisal, analysts will estimate the value of operating real estate holdings of a REIT by capitalizing the REIT's cash **net operating income** (NOI). This process first requires the calculation of a market-required rate of return, known as the capitalization rate ("cap rate"), based on the prices of comparable recent transactions that have taken place in the market.

$$\text{cap rate} = \text{NOI}_{\text{comps}}/\text{transaction price}_{\text{comps}}$$

NOI is the amount of income remaining after subtracting vacancy and collection losses and operating expenses (e.g., insurance, property taxes, utilities, maintenance, and repairs) from potential gross income. NOI is calculated before subtracting financing costs, depreciation, general and administrative expenses, and income taxes.

Note that in calculating cash NOI, we subtract non-cash rent. Non-cash rent is the difference between the average rent over the term of a lease contract (i.e., straight-line rent) versus the amount of cash rent actually received in a period. Further, in forecasting NOI for the next year, two adjustments are made to the current-period NOI: (1) the impact of acquisitions (if any) for the current year, which are not fully reflected in the current period's NOI, and (2) application of a growth rate.

In the following example, we show how to calculate NAVPS. First, estimated first-year NOI is capitalized using a market cap rate. Next, we add the value of other tangible assets and subtract the value of REIT's liabilities to find total net asset value. Net asset value divided by the number of outstanding shares gives us NAVPS.

Generally, deferred financing expenses, deferred tax assets, and goodwill are excluded from total assets so as to only include "hard economic assets." Similarly, debt and other liabilities are adjusted to their market values if the difference relative to book value is significant.

EXAMPLE: Computing NAVPS

Vinny Cestone, CFA, is undertaking a valuation of the Anyco Shopping Center REIT, Inc. Given the following financial data for Anyco, estimate NAVPS based on forecasted cash net operating income.

Select Anyco Shopping Center REIT, Inc., Financial Information (in $ millions)

Last 12 months' NOI	$80
Cash and equivalents	$20
Accounts receivable	$15
Total debt	$250
Other liabilities	$50
Non-cash rents	$2
Full-year adjustment for acquisitions	$1
Land held for future development	$10
Prepaid/other assets (excluding intangibles)	$5
Estimate of next 12 months' growth in NOI	1.25%
Cap rate based on recent comparable transactions	8.0%
Shares outstanding	15

Answer:

	Last 12 months' NOI	$80
–	Non-cash rents[1]	$2
+	Full-year adjustment for acquisitions[2]	$1
=	Pro forma cash NOI for last 12 months	$79
+	Next 12 months' growth in NOI (@1.25%/year)[3]	$1
=	Estimated next 12 months' cash NOI	$80
÷	Cap rate[4]	8.0%
=	Estimated value of operating real estate[5]	$1,000
+	Cash and equivalents[6]	$20
+	Land held for future development	$10
+	Accounts receivable	$15
+	Prepaid/other assets (excluding intangibles)	$5
=	Estimated gross asset value	$1,050

–	Total debt[7]	$250
–	Other liabilities	$50
=	Net asset value	$750
÷	Shares outstanding	15
=	Net asset value per share[8]	$50

Notes:

1. Non-cash rent (difference between average contractual rent and cash rent paid) is removed.
2. NOI is increased to represent full-year rent for properties acquired during the year.
3. Cash NOI is expected to increase by 1.25% over the next year.
4. Cap rate is based on recent transactions for comparable properties.
5. Operating real estate value = expected next 12-month cash NOI / 8% capitalization rate.
6. Add the book value of other assets: cash, accounts receivable, land for future development, prepaid expenses, and so on. Certain intangibles, such as goodwill, deferred financing expenses, and deferred tax assets, if given, are ignored.
7. Debt and other liabilities are subtracted to get to net asset value.
8. NAVPS = NAV/number of outstanding shares.

LOS 33.h: Describe the use of funds from operations (FFO) and adjusted funds from operations (AFFO) in REIT valuation.

Analysts calculate and use two measures: FFO and AFFO.

1. **Funds from operations (FFO)**. FFO adjusts reported earnings and is a popular measure of the continuing operating income of a REIT or REOC. FFO is calculated as follows:

	Accounting net earnings
+	Depreciation, amortization, impairments, and write-downs
–	Gains from sales of property
+	Losses from sales of property
=	Funds from operations

   Depreciation is added back under the premise that accounting depreciation often exceeds economic depreciation for real estate. Gains from sales of property are excluded because these are not considered to be part of continuing income.

2. **Adjusted funds from operations (AFFO)**. AFFO is an extension of FFO that is intended to be a more useful representation of current economic income. AFFO is also known as **cash available for distribution (CAD)** or **funds available for distribution (FAD)**.

   The calculation of AFFO generally involves beginning with FFO and then subtracting non-cash rent and maintenance-type capital expenditures and leasing

costs (such as improvement allowances to tenants or capital expenditures for maintenance).

	FFO (funds from operations)
–	Non-cash (straight-line) rent adjustment
–	Recurring maintenance-type capital expenditures and leasing commissions
=	AFFO (adjusted funds from operations)

Non-cash rent often arises due to straight-lining of rent: non-cash rent is the amount by which the average contractual rent over a lease period exceeds the rent actually paid. A lease calling for increasing rental rates over the lease term will have significant non-cash rent early in the lease term. Capital expenditures related to maintenance, as well as expenses related to leasing the space in properties, are subtracted from FFO because they represent costs that must be expended in order to maintain the value of the properties.

AFFO is considered a better measure of economic income than FFO because AFFO considers the capital expenditures that are required to sustain the property's economic income. AFFO also provides a better indicator for the sustainability of the REIT's dividend paying capacity. However, FFO is more frequently cited in practice, because AFFO relies more on estimates and is considered more subjective.

Comparison of Different Methods

Net asset value per share: The net asset value method of valuation can be used either to generate an absolute valuation or as part of a relative valuation approach. Note, however, that net asset value is an indication of a REIT's assets to a buyer in the private market, which can be quite different from the value public market investors would attach to the REIT. For this reason, there have historically been significant differences (i.e., premiums or discounts) between NAV estimates and the prices at which REITs actually trade.

PROFESSOR'S NOTE

Relative valuation using NAVPS is essentially comparing NAVPS to the market price of a REIT (or REOC) share. If, in general, the market is trading at a premium to NAVPS, a value investor would select the investments with the lowest premium (everything else held constant).

Advantages of Price Multiples

1. Multiples P/FFO and P/AFFO are globally accepted in evaluating shares.
2. Multiples allow comparisons with other investment alternatives.
3. FFO estimates are widely available from reputed data providers such as Bloomberg.
4. Multiples are evaluated in conjunction with other factors such as growth rates to allow for reconciliation of differences in multiples between different REITs.
5. Because leverage is not explicitly accounted for in FFO and AFFO, analysts need to adjust for leverage differences in relative value analysis.

Drawbacks of Multiples

1. Multiples do not capture the value of non-income-producing real assets (e.g., land held for development, vacant buildings).
2. FFO does not capture the impact of recurring capex-type maintenance expenditures needed to maintain the earning power of properties.
3. One-time gains and accounting charges distort FFO and AFFO and make comparisons between companies difficult.

Discounted cash flow approach: Dividend discount and discounted cash flow models of valuation are appropriate for use with REITs and REOCs, because these two investment structures typically pay dividends and thereby return a high proportion of their income to investors. DDM and DCF are used in private real estate in the same way that they are used to value stocks in general. For dividend discount models, an analyst will typically develop near-term, medium-term, and long-term growth forecasts and then use these values as the basis for two- or three-stage dividend discount models. To build a discounted cash flow model, analysts will generally create intermediate-term cash flow projections plus a terminal value that is developed using historical cash flow multiples.

LOS 33.i: Calculate and interpret the value of a REIT share using the net asset value, relative value (price-to-FFO and price-to-AFFO), and discounted cash flow approaches.

EXAMPLE: **Calculating the value of a REIT share**

Lucinda Crabtree, CFA, is an asset manager who is interested in diversifying the portfolio she manages through an investment in an office building REIT.

Crabtree wants to value the potential investment using four different approaches as of the end of 2023, as follows:

Approach 1: Net asset value

Approach 2: Price-to-FFO

Approach 3: Price-to-AFFO

Approach 4: Discounted cash flow

Selected REIT Financial Information

	All Amounts in $ Million
Estimated 12 months' cash net operating income (NOI)	$80
Last year's actual funds from operations (FFO)	$70
Cash and equivalents	$65
Accounts receivable	$35
Debt and other liabilities	$400
Non-cash rents	$5
Recurring maintenance-type capital expenditures	$15
Shares outstanding	10 million shares
Expected annual dividend next year (2024)	$5.00
Dividend growth rate in 2025 and 2026	2%
Dividend growth rate (from 2027 into perpetuity)	1%
Assumed cap rate	8%
Office subsector average P/FFO multiple	10×
Office subsector average P/AFFO multiple	14×
Crabtree's applicable cost of equity capital	9%
Risk-free rate	2%

Approach 1: Value of a REIT share using net asset value approach

The value per share for this REIT using net asset value valuation is computed as follows:

Estimated cash NOI	80
Assumed cap rate	8%
Estimated value of operating real estate (80 / 0.08)	1,000
Plus: cash + accounts receivable	100
Less: debt and other liabilities	400
Net asset value	700
Shares outstanding	10
NAV / share	$70

The REIT share value using the net asset value approach is thus $70. Note that no adjustment for non-cash rents was required in this case because we began with an estimate of cash NOI.

Approach 2: Value of a REIT share using price-to-FFO (P/FFO) approach

The value per share for this REIT using price-to-FFO valuation is computed as follows:

Funds from operations (FFO)	$70
Shares outstanding (millions)	10
FFO / share = $70 million / 10 million shares	$7

Applying the office subsector average P/FFO multiple of 10× yields a value per share of $7 × 10 = $70.

The REIT share value using the price-to-FFO approach is thus $70.

Approach 3: Value of REIT share using price-to-AFFO (P/AFFO) approach

Funds from operations (FFO)	$70
Subtract: non-cash rents	$5
Subtract: recurring maintenance-type capital expenditures	$15
Equals: AFFO	$50
Shares outstanding (million)	10
AFFO / share = $50 million / 10 million shares	$5
Property subsector average P/AFFO multiple	14×

Applying the office subsector average P/AFFO multiple of 14× yields a value per share of $5 × 14 = $70.

The REIT share value using the price-to-AFFO approach is thus $70.

Approach 4: Value of REIT share using discounted cash flow approach

	2024	2025	2026	2027
Dividends per share	$5.00	$5.10	$5.20	$5.25

Present value in 2026 of dividend stream beginning in 2027 = $5.25 / (0.09 – 0.01) = $65.63

These dividends are discounted at a rate of 9%.

value of a REIT share

= PV(dividends for Years 1 through n) + PV(terminal value at the end of Year *n*)

$= PV_{2024\ dividend} + PV_{2025\ dividend} + PV_{2026\ dividend} + PV_{2027\ and\ later\ dividends\ (terminal\ value)}$

$= \$5.00 / (1.09) + \$5.10 / (1.09)^2 + \$5.20 / (1.09)^3 + \$65.63 / (1.09)^3$

= $63.61

The REIT share value using the discounted cash flow approach is thus $63.61.

Note that the calculated value of a REIT share is likely to vary, sometimes greatly, depending on which of these approaches is used.

LOS 33.j: Explain advantages and disadvantages of investing in real estate through publicly traded securities compared to private vehicles.

We have already identified the advantages and drawbacks of investing in REITs versus private investment (i.e., direct investment) in real estate. We summarize them here.

Advantages of private investment relative to publicly traded real estate securities

1. Direct exposure to the real estate class.
2. Returns dictated by property performance.
3. Tax benefits (accelerated depreciation, timing option for capital gains).
4. Inflation hedge.
5. Illiquidity premium.
6. Control and ability to pursue diverse strategies.

7. Lower correlation with other asset classes (superior diversification).

Drawbacks of private investments relative to publicly traded securities

1. Illiquidity.
2. High fees and expenses.
3. High minimum investment.
4. Low transparency.
5. Fewer regulatory protections for investors.
6. Appraisals lag actual market values.
7. Higher returns derived from the use of leverage, but increases risk.

Advantages of investing in publicly traded securities

1. Tracks real estate asset class fundamentals over the long term.
2. High liquidity.
3. Professional management.
4. Inflation hedge.
5. Tax efficiency (for REITs).
6. Access to a (diversified) pool of assets.
7. Access to special sectors (e.g., data centers).
8. Low minimum investment.
9. Low entry/exit costs.
10. Regulatory protection for investors.
11. High transparency.
12. Limited liability.

Drawbacks of investing in publicly traded securities

1. Higher volatility.
2. Higher correlation with stocks (compared to the correlation of stocks with private real estate).
3. Dividends are taxed at high current income tax rates.
4. Poor governance/agency conflict.
5. Equity markets penalize high leverage.
6. Market prices differ from NAV.
7. REIT structure limits possible activities.
8. Compliance costs may be prohibitive for smaller companies.

MODULE QUIZ 33.2

1. Which of the following *least accurately* identifies one of the principal types of publicly traded real estate securities?
   A. Commingled real estate fund (CREF).
   B. Shares of real estate operating companies (REOC).
   C. Residential and commercial mortgage-backed securities (MBS).
2. Which of the following statements *most accurately* describes one of the advantages of investing in REITs?

A. REITs can pass on tax losses to their investors as deductions from their taxable income.
B. REITs have lower price and return volatility than a comparable direct investment in properties.
C. REITs provide a greater potential for diversification within the asset class.

3. Which of the following statements about the use of net asset value per share (NAVPS) in REIT valuation is *most accurate*?
   A. NAVPS is the difference between the accounting book values of a real estate company's assets and its liabilities, divided by shares outstanding.
   B. NAVPS is considered to be a superior measure of the net worth of a REIT's shares, compared with book value per share.
   C. NAVPS is exactly equal to the intrinsic value of REIT shares.
4. In the process of calculating adjusted funds from operations (AFFO) from funds from operations (FFO), an analyst is *most likely* to:
   A. add depreciation and amortization.
   B. subtract non-cash rent.
   C. add recurring maintenance-type capital expenditures and leasing commissions.
5. Which statement regarding approaches to REIT valuation is *least accurate*?
   A. AFFO includes a number of adjustments to FFO that result in AFFO approximating continuing cash earnings.
   B. P/AFFO is the most frequently used multiple in analyzing the REIT sector.
   C. Dividend discount models are appropriate for valuing REITs because REITs return most of their income to investors.

Use the following information to answer Questions 6 through 9.

Anna Ginzburg, CFA, is using the following information to analyze a potential investment in an industrial building.

Selected REIT Financial Information

	All Amounts in $ Million
Estimated 12 months' cash net operating income	$40
Funds from operations (FFO)	$30
Cash and equivalents	$30
Accounts receivable	$20
Debt and other liabilities	$250
Non-cash rents	$5
Recurring maintenance-type capital expenditures	$10
Shares outstanding	10 million shares
Expected annual dividend next year (2024)	$3.00
Dividend growth rate in 2025 and 2026	4%
Dividend growth rate (from 2027 into perpetuity)	3%
Assumed cap rate	8%
Office subsector average P/FFO multiple	12×
Office subsector average P/AFFO multiple	20×
Ginzburg's cost of equity capital	11%
Risk-free rate	2%

6. The value of Ginzburg's potential investment using a net asset value (NAV) approach is *closest* to:
   A. $30.
   B. $35.
   C. $40.

7. The value of Ginzburg's potential investment using a price-to-FFO approach is *closest* to:
   A. $30.
   B. $35.
   C. $40.
8. The value of Ginzburg's potential investment using a price-to-AFFO approach is *closest* to:
   A. $30.
   B. $35.
   C. $40.
9. The value of Ginzburg's potential investment using a discounted cash flow approach is *closest* to:
   A. $30.
   B. $35.
   C. $40.
10. Compared with other publicly traded shares, REITs are *most likely* to offer relatively low:
    A. yields.
    B. stability of income and returns.
    C. growth from reinvested operating cash flows.

KEY CONCEPTS

LOS 33.a

There are four basic forms of real estate investment: private equity (direct ownership), publicly traded equity (indirect ownership), private debt (direct mortgage lending), and publicly traded debt (mortgage-backed securities).

Real estate investments are heterogeneous, have high unit values, have high transaction costs, depreciate over time, are influenced by the cost and availability of debt capital, are illiquid, and are difficult to value.

Risks include changing business conditions, long lead times to develop property, cost and availability of capital, unexpected inflation, demographic factors, illiquidity, environmental issues, property management expertise, and the effects of leverage.

LOS 33.b

Reasons to invest in real estate include current income, capital appreciation, inflation hedge, diversification, and tax benefits. Real estate is less-than-perfectly correlated with the returns of stocks and bonds; thus, adding real estate to a portfolio can reduce risk relative to the expected return.

While factors such as GDP growth, population growth, wage growth, taxes, and regulations affect all property types, factors such as changing supply routes, advances in logistics, trade, and transportation are unique to industrial properties.

LOS 33.c

Commercial property types, and the demand for each, are driven by the following:

- Office—Job growth.
- Industrial—The overall economy.
- Retail—Consumer spending.

- Multifamily—Population growth.

LOS 33.d

Investors perform due diligence to confirm the facts and conditions that might affect the value of the transaction. Due diligence can be costly, but it lowers risk of unexpected legal and physical problems. Due diligence involves reviewing leases, confirming expenses, performing inspections, surveying the property, examining legal documents, and verifying compliance.

LOS 33.e

Appraisal-based indices calculate return as current yield (from NOI) plus price appreciation (adjusted for capital expenditures). Transaction-based indices are based on (1) repeat sales, (2) a hedonic model, or (3) public securities. Appraisal-based indices tend to lag transaction-based indices and appear to have lower volatility and lower correlation with other asset classes.

LOS 33.f

The main types of REITs are:

- Equity REITs, which take ownership stakes in income-producing property.
- Mortgage REITs, which invest primarily in mortgages, mortgage securities, or loans that use real estate as collateral.

Advantages of REITs include:

- Superior liquidity.
- Transparency.
- Access to premium properties.
- Active professional management.
- Greater potential for diversification.
- Exemption from taxation.
- Earnings predictability and high yield.

Disadvantages of REITs include:

- Limited potential for income growth.
- Forced equity issuance.

LOS 33.g

Net asset value per share (NAVPS) is the (per-share) amount by which a REIT's assets exceed its liabilities, using current market value of REIT real estate investments rather than accounting or book values. The REIT or REOC portfolio of operating real estate investments can be valued by capitalizing cash net operating income. Adding other assets and subtracting other liabilities yields net asset value.

LOS 33.h

	Accounting net earnings
+	Depreciation, amortization, impairments, and writedowns
–	Gains (losses) from sales of property
=	Funds from operations (FFO)

	FFO (funds from operations)
–	Non-cash (straight-line) rent adjustment
–	Recurring maintenance-type capital expenditures and leasing commissions
=	AFFO (adjusted funds from operations)

LOS 33.i

Price-to-FFO approach:

	Funds from operations (FFO)
÷	Shares outstanding
=	FFO / share
×	Sector average P/FFO multiple
=	NAV / share

Price-to-AFFO approach:

	Funds from operations (FFO)
–	Non-cash rents
–	Recurring maintenance-type capital expenditures
=	AFFO
÷	Shares outstanding
=	AFFO / share
×	Property subsector average P/AFFO multiple
=	NAV / share

Discounted cash flow approach:

$$\text{value of a REIT share} = \text{PV(dividends for Years 1 through } n) + \text{PV(terminal value at the end of Year } n)$$

LOS 33.j

Relative to publicly traded securities, private investment provides direct exposure to the real estate class, returns dictated by property performance, tax benefits, inflation hedge, illiquidity premium, control, and lower correlation with other asset classes. However, private investments suffer from illiquidity, high fees and expenses, high minimum investment, low transparency, fewer regulatory protections for investors, and higher risk due to leverage.

Publicly traded securities are attractive because they provide high liquidity, professional management, tax efficiency (REITs), diversification, low minimum investment, low entry/exit costs, regulatory protection for investors, and high transparency. However, they suffer from higher volatility, higher correlation with stocks, and agency conflict.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 33.1

1. **B** Of the three investment choices, REITs are the most liquid because the shares are actively traded. Also, REITs provide quick and easy diversification across many properties. Neither the direct property investment nor the mortgage participation is liquid, and significant capital would be required to diversify the investments. (LOS 33.a)

2. **C** Risks include changing business conditions, long lead times to develop property, cost and availability of capital, unexpected inflation, demographic factors, illiquidity, environmental issues, property management expertise, and the effects of leverage. Private real estate investment provides higher diversification benefits (in a portfolio with financial assets such as stocks and bonds) than REITs. (LOS 33.a)

3. **A** Residential real estate (i.e., an apartment building) purchased with the intent to produce income is considered commercial real estate property. Cell towers are generally excluded from commercial property category. (LOS 33.a)

4. **C** An all-cash transaction eliminates financial leverage and lowers risk. Inflation risk is typically lower with a real estate investment, but the risk is not totally eliminated. If interest rates rise, non-leveraged property values are still impacted. Investors require higher returns when rates rise. Resale prices also depend on the cost and availability of debt capital. (LOS 33.a)

5. **C** Demand for multifamily properties depends on population growth, especially in the age demographic that typically rents apartments. (LOS 33.c)

6. **A** NCREIF property index (NPI) uses a value-weighted return that is calculated using *appraisal* data. The index allows comparison of performance of a real estate asset class with that of other asset classes. Because it is an appraisal-based index and not based on actual transactions, it suffers from appraisal lag. (LOS 33.e)

7. **C** The public records should be searched for outstanding liens filed by contractors involved in the renovation. An existing lien can result in legal problems for the purchaser and the lender. A survey will not identify outstanding liens. A survey confirms the property boundaries and identifies any easements. (LOS 33.d)

8. **B** Appraisal-based indices tend to lag transaction-based indices because actual transactions occur before appraisals are performed (appraisals are based on transaction data). Appraisal-based indices, not transaction-based indices, appear to have lower correlations with other asset classes. (LOS 33.e)

9. **A** After growth in the GDP, the most important factor driving demand for hotel rooms is job creation, because business and leisure travel are closely tied to the size of the workforce. More important to the value of a storage REIT than retail sales growth is population growth. More important to the value of an office REIT than population growth is job creation. (LOS 33.b)

Module Quiz 33.2

1. **A** A commingled real estate fund (CREF) is an example of a private real estate investment, not a publicly traded security. The three principal types of publicly traded real estate securities available globally are real estate investment trusts (REITs), real estate operating companies (REOCs), and residential and commercial mortgage-backed securities (MBS). (LOS 33.f)

2. **C** Compared to direct investment in real estate, a REIT's portfolio investment includes a number of geographically dispersed properties allowing for diversification benefits within the real estate asset class. REITs and REOCs usually cannot pass on tax losses to their investors as deductions from taxable income. Because REIT prices and returns are determined by the stock market, the value of a REIT is more volatile than its appraised net asset value. (LOS 33.f)

3. **B** NAVPS is the difference between a REIT's assets and its liabilities, using current market values instead of accounting book values and dividing by the number of shares outstanding. NAVPS is a superior measure of the net worth of a REIT, compared with book value per share, which is based on historical cost values. NAV is the largest component of the intrinsic value of a REIT; however, other factors, such as the value of non-asset-based income streams, the value added by management, and the value of any contingent liabilities, also contribute to intrinsic value. (LOS 33.g)

4. **B** To calculate AFFO, we begin with FFO and then deduct non-cash rent, maintenance-type capital expenditures, and leasing commissions. (LOS 33.h)

5. **B** FFO has some shortcomings, but because it is the most standardized method of measuring a REIT's earnings, P/FFO is the most commonly used multiple in analyzing REITs. AFFO is used as a convenient proxy for a "cash flow" multiple because AFFO approximates cash earnings. Dividend discount models are appropriate methods for valuing REITs because REITs return a significant portion of their income to their investors and tend to be high-dividend payers. (LOS 33.h)

6. **A** The value per share for this REIT using net asset value valuation is calculated as follows:

Estimated cash NOI	40
Assumed cap rate	8%
Estimated value of operating real estate (40 / 0.08)	500
Plus: cash + accounts receivable	50
Less: debt and other liabilities	250
Net asset value	300
Shares outstanding	10
NAV / share	$30

The REIT share value using the net asset value approach is $30. (LOS 33.i)

7. **B** The value per share for this REIT using price-to-FFO valuation is computed as follows:

Funds from operations (FFO)	$30
Shares outstanding (millions)	10
FFO / share = $30 million / 10 million shares	$3

Applying the office subsector average P/FFO multiple of 12× yields a value per share of:

$\$3 \times 12 = \$36$

The REIT share value using the price-to-FFO approach is $36. (LOS 33.i)

8. **A** The value per share for this REIT using a price-to-AFFO valuation is calculated as follows:

Funds from operations (FFO)	$30
Subtract: non-cash rents	$5
Subtract: recurring maintenance-type capital expenditures	$10
Equals: AFFO	$15
Shares outstanding	10 million
AFFO / share = $15 million / 10 million shares	$1.50
Property subsector average P/AFFO multiple	20×

Applying the office subsector average P/AFFO multiple of 20× yields a value per share of $\$1.50 \times 20 = \$30$.

The REIT share value using the price-to-AFFO approach is $30. (LOS 33.i)

9. **C** The value per share for this REIT using a discounted cash flow valuation is calculated as follows:

	2024	2025	2026	2027
Dividends per share	$3.00	$3.12	$3.24	$3.34

present value in 2026 of dividends stream beginning in 2027 = $3.34 / (0.11 – 0.03) = $41.78

present value of all dividends, when discounted at a rate of 11%

$= PV_{2024\ dividend} + PV_{2025\ dividend} + PV_{2026\ dividend} + PV_{(terminal\ value)}$
$= \$3.00 / (1.11) + \$3.12 / (1.11)^2 + \$3.24 / (1.11)^3 + \$41.78 / (1.11)^3$
$= \$38.15$

The REIT share value using the discounted cash flow approach is $38.15. (LOS 33.i)

10. **C** When we compare REITs to other kinds of publicly traded shares, REITs offer above-average yields and stable income and returns. Due to their high income-to-payout ratios, REITs have relatively low potential to grow by reinvesting operating cash flows. (LOS 33.j)

READING 34

HEDGE FUND STRATEGIES

EXAM FOCUS

This reading describes some of the most important categories of hedge fund strategies, examines their investment characteristics, and explains how these strategies are implemented. Understand the investment characteristics and implementation approach for six major hedge fund strategy categories. Be able to interpret the model for understanding the risk exposures of each of these strategies.

MODULE 34.1: OVERVIEW OF HEDGE FUND STRATEGIES

Video covering this content is available online.

Introduction

Hedge funds represent an important subgroup of alternative investment opportunities.

Key features that distinguish hedge funds from traditional investments include:

- Lower regulatory and legal constraints.
- Flexibility to use short selling and derivatives.
- A larger investment universe.
- Aggressive investment exposures.
- Comparatively free use of leverage.
- Liquidity constraints for investors.
- Lack of transparency.
- Higher cost structures.

The key question is whether the high expense levels of hedge funds are justified by the return and diversification benefits that hedge funds are intended to deliver.

Some asset managers seek out hedge funds as a source of alpha, which tends to be elusive. Others view hedge funds as a way to access top investing talent.

LOS 34.a: Discuss how hedge fund strategies may be classified.

Classifications of Hedge Fund Strategies

Hedge fund strategies are categorized based on the kinds of securities they invest in, the trading approach they use, and the kinds of risk exposures taken.

There are numerous ways to classify hedge funds, but in this topic review, we use the following six strategy categories:

1. *Equity related.* These fund strategies focus on stocks, and hence the primary source of risk is equity risk. There are several subtypes, including long/short equity, dedicated short bias, and equity market neutral.
2. *Event driven.* These strategies relate to corporate actions such as governance activities, mergers, acquisitions, bankruptcies, and other major business events. The main dangers of these strategies are event risks—the possibility that outcomes will not unfold as expected (examples include: the failure of a merger, credit downgrades, or bankruptcy). In the following LOSs, the event-driven hedge fund strategies that we will examine are merger arbitrage and distressed securities.
3. *Relative value.* These hedge fund strategies seek to profit from the price differentials between related securities. For debt securities, the valuation differences being exploited span securities with different credit quality or liquidity. Two relative value strategies that will be considered further are fixed-income arbitrage and convertible bond arbitrage.
4. *Opportunistic.* These strategies employ a top-down approach, often span multiple asset classes, and vary with market conditions. The two opportunistic strategies that will be considered here are global macro and managed futures.
5. *Specialist.* These strategies generally require specialized market expertise or knowledge. Often the risks of these strategies arise due to exposure to specific sectors or unusual securities. The two such specialist strategies we will consider in this topic review are volatility strategies and reinsurance strategies.
6. *Multi-manager.* These strategies use other hedge fund strategies as building blocks, combining different strategies together and rebalancing exposures over time. The two types of multi-manager hedge funds we will consider are multi-strategy funds and funds-of-funds.

MODULE QUIZ 34.1

1. A convertible bond arbitrage strategy is *most likely* to be classified as a(n):
   A. specialist strategy.
   B. event-driven strategy.
   C. relative value strategy.
2. A managed futures hedge fund strategy is *most likely* to be classified as a(n):
   A. opportunistic strategy.
   B. specialist strategy.
   C. relative value strategy.

MODULE 34.2: EQUITY, EVENT-DRIVEN, AND RELATIVE VALUE STRATEGIES

Video covering this content is available online.

LOS 34.b: Discuss investment characteristics, strategy implementation, and role in a portfolio of equity-related hedge fund strategies.

Equity-Related Hedge Fund Strategies

Equity-related hedge fund strategies focus primarily on stock markets, and their risk primarily stems from stocks. Subtypes of the equity-related category include: long/short equity, dedicated short bias, and equity market neutral.

Long/Short Equity

Long/short (L/S) equity hedge funds are straightforward to understand. The fund manager purchases (takes long positions in) stocks that they think will rise in value, and sells (takes short positions in) stocks that they believe will fall in value.

Investment Characteristics

When long/short equity managers combine long and short positions, the resulting portfolio has a beta (i.e., market exposure) equal to the weighted sum of the positive and negative betas of the various long and short positions.

L/S equity hedge funds generally do not seek to *eliminate* market exposure entirely. Rather, L/S funds will typically have a 40% to 60% net long exposure, which is beneficial considering that markets generally trend upward over time.

L/S equity managers typically aspire to provide returns comparable to those of a long-only fund, but with half the amount of standard deviation.

Strategy Implementation

Because successful implementation of a long/short equity strategy requires managers to identify overpriced and underpriced stocks, the majority of L/S equity funds take a sector-specific focus, choosing securities from a particular industry that they are familiar with. They may also use index funds to achieve a desired exposure. L/S funds that are comparatively market neutral may need to use leverage to achieve worthwhile returns.

Role in a Portfolio

The goal of most L/S equity managers is to derive alpha from long and short positions in individual stocks, while also benefitting from a moderate overall long exposure.

When considering an investment in an L/S equity fund, an analyst should weigh whether the investment is worthwhile given the potentially high fees. (Taking a traditional long-only equity position may be a more efficient way to achieve a comparable beta exposure.)

Dedicated Short Selling and Short-Biased

As the name suggests, **dedicated short-selling** funds seek out securities that are overpriced in order to sell them short. **Short-biased** managers use a similar strategy, except that the short position is somewhat offset by a long exposure.

One major challenge of being a short seller is that market prices generally rise over time, which creates a tendency towards negative returns for shorts.

A notable subset of short selling is **activist short selling**, in which the fund manager not only takes a short position in a stock, but also presents research that contends that the stock is overpriced.

Investment Characteristics

Managers using short-biased strategies or dedicated short-selling strategies seek to produce a negative correlation with conventional securities. Compared to other hedge fund strategies, expectations of return for short strategies tend to be lower. Compared to L/S equity hedge funds (which feature offsetting beta exposures) short strategy funds tend to have greater volatility.

Strategy Implementation

Hedge fund managers go "short" a security by borrowing the security and selling it at the current market price. The trade will be profitable if the manager is later able to repurchase the same security at a lower price in order to return it to the lender.

The challenging part of profitably shorting stocks is correctly identifying securities that will lose value. Managers will use a bottom-up approach to identify firms with unprofitable business models, bad management, too much debt, declining market segments, or even deceitful accounting.

A dedicated short seller does not take on any long stock position; rather, they carefully select stocks for a pure short exposure, typically 60% to 120% short. A dedicated short manager who wishes to temper the fund's market exposure will typically do this by holding cash.

Short-biased managers have a similar strategy; however, they may also take on some long exposure, while remaining net short, often 30% to 60% net short.

For both dedicated short sellers and short-biased managers, relatively little leverage is used.

Role in a Portfolio

The primary goal of dedicated short-selling and short-biased funds is to produce returns that are uncorrelated (or negatively correlated) with the return of conventional portfolio assets. When successful, these negative correlations provide a diversification benefit to a portfolio.

However, this goal of negative correlation comes at a cost: expected returns for short strategies are relatively low.

Equity Market Neutral

Equity market-neutral (EMN) strategies seek to attain a near-zero overall exposure to the stock market. They do this by taking long and short positions in various equities; the weighted betas of these positions should sum to zero. The alpha of EMN strategies is intended to be derived from taking positions in securities that are temporarily mispriced.

Investment Characteristics

The overall goal of EMN funds is to create a portfolio that not only generates alpha, but also is relatively immune to movements in the overall market. Not surprisingly, without beta exposure, the returns of EMN funds tend to be modest. On the other hand, EMN funds can offer significant diversification and low volatility.

Strategy Implementation

EMN fund managers take long positions in particular stocks thought to be temporarily undervalued and short positions in stocks seen as overvalued. When mean reversion eventually occurs, alpha should result.

While some EMN managers (known as discretionary managers) rely on intuition, it is more common for EMN managers (called quantitative managers) to rely on a fixed set of rules to identify trade opportunities.

Because EMN deliberately hedges away market beta, leverage must generally be applied in order to achieve acceptable levels of return.

Popular subtypes of EMN funds include:

- **Pairs trading.** Two stocks with similar characteristics but that are respectively overvalued and undervalued are identified. The co-integrated trading prices of pairs are monitored, and unusual divergence is exploited.
- **Stub trading.** This EMN strategy involves going long and short shares of a subsidiary and its parent company. Generally the positions taken correspond to the percentage of the subsidiary owned by the parent.
- **Multi-class trading.** This strategy entails going long and short relatively mispriced share classes of the same firm—for example, non-voting and voting shares. Profits will arise as the prices of these shares revert to their historical relative valuations.

Aside from investments in stocks, other instruments that can be used to produce a state of zero beta include options, stock index futures, and other derivatives.

Role in a Portfolio

EMN portfolios attempt to produce alpha without taking market beta risk: The composition of EMN funds allows these funds to produce less volatility than funds that rely on beta as a source of return. EMN strategies are particularly successful when markets are volatile and performing poorly.

LOS 34.c: Discuss investment characteristics, strategy implementation, and role in a portfolio of event-driven hedge fund strategies.

Event-Driven Strategies

Event-driven hedge fund strategies are those that attempt to profit from accurately predicting the outcome of corporate events, such as bankruptcies, mergers, restructurings, acquisitions, et cetera. To do this, these funds take positions in securities of these corporations or in related derivatives. A **soft-catalyst event-driven** approach is an investment made *before* an event has been announced. A **hard-catalyst event-driven** approach is an investment made *after* a corporate event has been announced; this strategy seeks to take advantage of security prices that have not fully adjusted. Soft-catalyst investing is generally more volatile (and, thus, riskier) than a hard-catalyst approach.

The main risk that impacts event-driven strategies is **event risk**, reflecting the possibility that the outcome of the event will not be the one anticipated. For example, a merger arbitrage hedge fund that anticipates that a particular merger will be successful can suffer a large loss if the merger fails.

In this topic review, we will consider two types of event-driven hedge fund strategies in detail: merger arbitrage and distressed securities.

Merger Arbitrage

Merger arbitrage strategies are investment schemes that attempt to earn a return from the uncertainty that exists in the market during the time between an acquisition being announced and when the acquisition is completed.

Hedge fund managers in the merger arbitrage space profit by correctly anticipating the outcome of various deals. One way to view merger arbitrage is to compare it to writing (selling) insurance on an acquisition. If the acquisition is completed as planned, the hedge fund earns an insurance premium. If the transaction fails, however, the hedge fund stands to lose money—analogous to an insurance company making a payout.

Investment Characteristics

In the case of a merger deal that fails, the price movements that originally occurred when the merger was announced will reverse: the price of the target will fall, and the price of the acquirer will rise. A hedge fund that had taken a position based on the merger succeeding is likely to suffer a significant downside when the deal fails unexpectedly; perhaps on the order of a 40% loss. This kind of potential outcome gives merger arbitrage a significant left-tail risk.

Compared to typical hedge fund strategies, merger arbitrage tends to be more liquid.

Strategy Implementation

In the most common merger arbitrage scenario, the portfolio manager takes positions in the securities of the companies involved, with the expectation of a successful deal. For example, in a stock-for-stock deal, the hedge fund manager will typically purchase the stock of the target company and *short* the stock of the acquiring company in anticipation of profiting when the deal is completed.

Less commonly, the hedge fund manager may have the opinion that the merger will fail (for example, the deal might be blocked by the government because it would stifle competition). In this scenario, the fund would take the *opposite* positions to those just described.

In order to generate a worthwhile level of return, hedge funds pursuing a merger arbitrage strategy will typically apply 300% to 500% leverage in pursuit of low-double-digit returns.

One specific variety of merger arbitrage involves cross-border mergers and acquisitions (M&A) where two countries and two regulatory authorities are involved. Such mergers are seen as more risky.

Role in a Portfolio

The Sharpe ratios of merger arbitrage strategies tend to be high, as these strategies usually produce relatively steady returns. However, despite these generally steady returns, there is significant left-tail risk.

Distressed Securities

Hedge funds that pursue a **distressed securities** strategy take positions in the securities of companies that are experiencing financial difficulties, including firms that are in bankruptcy or near bankruptcy. Firms may find themselves in this position for a number of reasons, including too much leverage, difficulty competing in their sector, or accounting irregularities. The securities of such a firm will often trade at greatly depressed prices.

Compounding the discounting of the securities of distressed firms is the fact that institutions such as insurance companies and banks are often not permitted to hold non-investment-grade securities. The resultant selling of downgraded distressed securities can create significant pricing inefficiencies and can open up opportunities for hedge funds seeking profit.

When a firm is liquidated, the assets of the company are sold, and the various investors are paid back *sequentially* based on their seniority: senior secured debtholders are paid first, then holders of less-senior secured debt, unsecured debt, convertible debt, preferred stock, and lastly common stock.

As an alternative to liquidation, a firm may instead be able to reorganize, which may involve renegotiating the company's liabilities. Holders of debt may be asked to exchange that debt for new equity or to agree to an extension of the maturity.

Investment Characteristics

Relative to the various event-driven hedge fund strategies, returns of a distressed securities investing strategy tend to be somewhat greater—though generally with larger variability of outcomes.

The **lock-upperiods** for investors in event-driven hedge funds tend to be comparatively long (often allowing no redemptions for the first two years). This reflects the extended period of time it can take to value and exit a distressed security investment.

Strategy Implementation

Distressed investing can take different forms. Some managers may make only a passive investment in the distressed securities. Other managers will attempt to acquire the majority of a certain class of security in order to take creditor control during bankruptcy.

Successful distressed securities investing requires a particularly broad range of skills in order to navigate the various legal aspects of the strategy, including the bankruptcy and reorganization proceedings.

While shorting distressed securities is a possibility, the majority of distressed investing takes the form of long investments.

Distressed investing generally makes low use of leverage.

Role in a Portfolio

Distressed securities investing involves moderately high levels of illiquidity due to the nature of the assets being purchased. The returns of distressed securities investing are on-average higher than those of other event-driven strategies, though they are unpredictable and sensitive to declines in the overall market.

LOS 34.d: Discuss investment characteristics, strategy implementation, and role in a portfolio of relative value hedge fund strategies.

Relative Value Hedge Fund Strategies

As the name suggests, **relative value** strategies attempt to exploit valuation differences between securities. The most common securities used in relative value strategies are hybrid convertible debt, though other types of fixed-income securities may be used.

When successful, relative value strategies will earn various premiums over time, including liquidity, credit, and volatility premiums, reflecting differences in liquidity or credit quality between securities. However, during turbulent times, losses can occur.

Two relative value hedge fund strategies that we will consider in detail are fixed-income arbitrage and convertible bond arbitrage.

Fixed-Income Arbitrage

The idea behind **fixed-income arbitrage** strategies is to take advantage of temporary mispricing of fixed-income instruments, by going long undervalued securities, and going short comparatively overvalued securities. Other idiosyncrasies that might be exploited include yield curve kinks or anticipated changes in the shape of the yield curve.

The fixed-income instruments can be of various types, including consumer debt, bank loans, corporate bonds, or sovereign bonds. Hedge fund managers take positions in these securities in the belief that prices will revert toward their fair values.

Fixed-income arbitrage strategies often make use of significant leverage, in order to produce sufficient levels of return.

Strategy Implementation

Two subtypes of fixed-income arbitrage strategies that we will consider in further detail are (1) yield curve trades and (2) carry trades.

- **Yield curve trades.** In this strategy, the hedge fund manager forms a view of how the shape of the yield curve will evolve over time based on macroeconomic forecasts. The portfolio manager then makes long and short investments in fixed-income instruments in order to profit from the anticipated yield curve steepening or flattening. The portfolio manager will profit when the prices of the long securities rise and the short securities fall. If the positions taken are in securities of different firms, then liquidity, credit, and interest rate risks will be present. For

positions in securities of the same issuer, interest rate movements would be the main source of risk.

- **Carry trades.** In a carry trade, the portfolio manager shorts a low-yielding security and goes long a high-yielding security. The source of return here is twofold; first, from the yield differential, and second from the price changes as mean reversion occurs.

Investment Characteristics

Because fixed-income securities tend to be priced efficiently, the amount of profit that can be earned through fixed-income arbitrage is somewhat limited. As a result, substantial leverage is often applied to fixed-income arbitrage strategies. (400% leverage is not uncommon; even 1500% leverage is not unheard of.)

The liquidity of fixed-income arbitrage depends on the particular strategy employed and the kinds of fixed-income instruments used. Strategies involving U.S. Treasuries are very liquid, while strategies that make use of, for example, mortgage-backed securities or foreign instruments will be more difficult to convert to cash.

Role in a Portfolio

Return distributions for a fixed-income arbitrageur tend to be similar to the returns from writing puts. If the trade unfolds as expected, the investor will earn a return from the spread narrowing, plus a return from positive carry. However, if the spread between the two instruments widens unexpectedly, the use of leverage may result in the return to the investor becoming quite negative.

One drawback to fixed-income arbitrage strategies is that their highly leveraged nature can cause modest price volatility to lead to a domino effect of margin calls and deleveraging. For example, the Asian Financial Crisis of 1997 and the Russian Ruble Crisis of 1998 led to the collapse of the renowned hedge fund Long-Term Capital Management.

Convertible Bond Arbitrage

Convertible bonds are fixed-income debt securities that make regular coupon payments but can additionally be exchanged for a prearranged number of common stock shares. The bond-to-stock conversion is at the bondholder's discretion, though it is only permitted at certain points in the bond's life.

One way to view convertible bonds is as a regular bond, plus a long call option on the corresponding stock.

Analyzing convertible bonds can be complex due to impacts from a number of factors.

The goal of **convertible bond arbitrage** strategies is generally to profit from purchasing the implied volatility of convertible bonds—which is often underpriced. To accomplish this without taking on excess potential for loss, convertible bond arbitrageurs will take other positions in an attempt to hedge the delta and gamma risk of the convertible bond.

Investment Characteristics

Convertible bond arbitrage managers encounter two primary sources of liquidity issues; first, because the strategy requires the manager to short sell the underlying equity, and second, because the fixed-income instruments that are invested in are often complex niche products.

Strategy Implementation

Convertible arbitrage strategies generally exploit the low implied volatilities exhibited by the options within convertible instruments, compared to the historical volatilities of the equities that underlie the option. The challenge is to hedge away the other sources of risk that are embedded in the convertible security including market risk, interest rate risk, and the credit risk of the bond issuer.

When a convertible bond's current conversion price is well above the current stock price, the out-of-the-money call (delta closer to zero) will lead to the convertible bond behaving much like a straight bond, because conversion to stock is unlikely.

Conversely, when a convertible bond's current conversion price is well below the current stock price, the in-the-money call (with delta near one) will cause the convertible bond to behave much like the corresponding stock because conversion is likely.

Significant amounts of leverage are typically applied in implementing convertible bond strategies. For example, a three times long bond exposure and two times short equity exposure might be used. (The smaller short equity exposure stems from delta hedging the short stock exposure according to the delta of the embedded call of the convertible.)

Role in a Portfolio

Convertible arbitrage strategies perform best during periods of normal market conditions: when liquidity is available, when volatility is modest, and when there is a good selection of convertible bonds being issued. Convertible arbitrage may not perform well in periods of illiquidity or acute credit weakness.

MODULE QUIZ 34.2

1. Considering the following equity hedge fund strategies, the strategy that is *most likely* to apply relatively high levels of leverage is a(n):
   A. equity market neutral strategy.
   B. dedicated short strategy.
   C. short-biased strategy.
2. An equity-related hedge fund strategy with gross exposures of 80% long and 35% short is *most likely* to be classified as a:
   A. dedicated short strategy.
   B. short-biased strategy.
   C. long/short equity strategy.
3. Relative to other hedge fund strategies, equity market neutral strategies are *most likely* to:
   A. exhibit relatively modest returns.
   B. be vulnerable to periods of market weakness.
   C. earn return from alpha and beta risk.
4. An investment in distressed securities is *most likely* to be characterized by a:
   A. long bias.

B. high level of liquidity.
C. large amount of leverage.

5. In a sequential payoff during a liquidation, the security holder that is *most likely* to be paid off first is the holder of:
   A. junior secured debt.
   B. convertible debt.
   C. preferred stock.
6. In implementing a convertible arbitrage strategy, a portfolio manager is *most likely* to take a position that is:
   A. long convertible bonds and short equity.
   B. long straight bonds and short convertible bonds.
   C. long convertible bonds and short straight bonds.

MODULE 34.3: OPPORTUNISTIC, SPECIALIST, AND MULTI-MANAGER STRATEGIES

Video covering this content is available online.

LOS 34.e: Discuss investment characteristics, strategy implementation, and role in a portfolio of opportunistic hedge fund strategies.

Opportunistic Hedge Fund Strategies

Opportunistic hedge fund strategies are a broad class of investing approaches that attempt to extract profits using a wide range of techniques in a broad range of securities. Rather than being focused on individual securities, these strategies take a top-down approach to make macro investments on a global basis across regions, sectors, and asset classes.

The returns of opportunistic hedge fund strategies may be impacted by market cycles, global developments, and international interactions. The risks will depend on the particular strategy and asset classes involved.

Opportunistic hedge fund strategies can be based on a number of broad techniques. **Technical analysis** uses past price changes to forecast future price movements. Strategies based on **fundamentals**, on the other hand, attempt to analyze security prices, markets, sectors, and the relationships between markets, using economic data as the input.

Managers using a **systematic** implementation of their strategies employ computer algorithms and rules to determine which trades to make. Managers using a **discretionary** process instead use their instinct to determine when to trade.

We will consider two opportunistic strategies in detail: global macro and managed futures.

Global Macro Strategies

Managers of **global macro** strategy funds attempt to make correct assessments and forecasts of various global economic variables including inflation, currency exchange rates, yield curves, central bank policies, and the general economic health of different countries.

Global macro managers use a broad range of security types and global asset classes to take positions on these views.

A global macro manager that can identify a global trend early and take a position can profit.

Investment Characteristics

Global macro managers can take positions that are either **directional** (e.g., go long stocks that are anticipated to benefit from expected interest rate hikes, and short stocks that will be disadvantaged), or **thematic** (e.g., buy stocks of firms that will benefit from forthcoming free trade deals).

Unlike some hedge fund strategies, low-volatility mean-reverting markets are not generally favorable for global macro returns.

Because global macro managers take investment positions based on their predictions of the future, there is significant potential for unsuccessful investments when global economies do not behave as expected or if unanticipated risks emerge. As a result, the returns of global macro funds tend to be uneven and volatile.

Strategy Implementation

Global macro strategies are generally based on top-down analysis, beginning with analysis of the global economy, then macro trends within economies, and so on, in order to identify potential opportunities.

Different global macro managers are likely to implement their strategies using very different techniques; for example, technical analysis vs. fundamental analysis, or discretionary implementation versus systematic implementation.

One commonality between global macro funds is that most tend to apply leverage, often representing 600% or 700% of fund assets.

A manager making directional predictions will generally use fundamental information to determine whether a particular asset is undervalued or overvalued on a historical basis. By contrast, a manager using a relative value strategy will seek out securities that are overvalued or undervalued *compared to one another*.

Global macro managers tend to use discretionary approaches more than do managed futures managers.

Role in a Portfolio

When added to a portfolio of traditional assets, a global macro hedge fund can add not only alpha, but also portfolio diversification.

A global macro manager will attempt to anticipate changes before other market participants and then will take a corresponding position and wait for the market to come around. For example, some global macro managers were able to anticipate the United States' subprime mortgage crisis well before the housing bubble began to collapse in 2007.

This contrarian tendency can make an allocation to global macro strategies especially advantageous.

During times of market stress, global macro funds have historically delivered right-tail skewed returns, which is beneficial from a portfolio diversification perspective. However, this behavior cannot always be relied upon, and such diversifying outcomes are not always realized.

Managed Futures

Hedge funds that pursue a **managed futures** strategy take long and short positions in a variety of derivatives contracts including futures, forwards, options on futures, swaps, and sometimes currencies and commodities.

Managed futures strategies can be as simple as trading index futures on a particular sector, or it can involve very exotic contracts such as futures on the weather.

Investment Characteristics

Managed futures funds do not buy and sell assets; rather, they enter into derivatives contracts in order to gain the desired exposures.

Because of the mechanics of futures contracts (requiring only a small amount of upfront collateral), managed futures funds can easily apply great amounts of leverage. Typically, a fund will use perhaps ⅛ of its capital as collateral on futures contracts. The rest of its capital will be invested in some highly liquid security (such as short-term government bonds) that can also serve as collateral for the futures clearinghouse.

Managed futures funds are extremely liquid, because futures contracts themselves are highly liquid: they trade globally and continuously. Taking long or short futures positions allows a hedge fund manager easy access to exposures across a range of asset classes.

One downside of the popularity of managed futures strategies is that **crowding** has occurred: many market participants pursue the same trades and use similar signals, resulting in a "herding effect" and execution slippage.

Strategy Implementation

There are a number of ways to implement managed futures strategies.

In perhaps the most popular method, **time-series momentum (TSM)** strategies, portfolio managers simply follow the trend: they buy securities that have been rising in price and sell securities that have been trending downward.

Another similar methodology is **cross-sectional momentum (CSM)** strategies, which is carried out within a particular asset class (a *cross section* of assets). Again, the securities rising fastest are purchased while falling securities are shorted.

The high liquidity of futures contracts allows hedge fund managers to pursue a wide selection of trading strategies. Generally, portfolio managers will rely on a signal trigger—most often based on volatility or momentum—to prompt a trade.

In addition to using trade signals, portfolio managers will also have rules for closing a position. Exit methodologies can be based on:

- Price targets.

- Momentum reversal.
- Time.
- Trailing stop-loss.
- Or, some combination of these approaches.

Managed futures investors seek to develop rules and signals that are profitable not only using historical data but also in real-world use. However, the more portfolio managers that use similar signals, the less effective these signals will become.

The size of positions taken are usually based on factors such as correlation and volatility.

Role in a Portfolio

Perhaps the most appealing feature of managed futures is their interaction with other investments. Overall, managed futures have very little correlation with traditional equity and fixed-income assets. The result is that when added to a portfolio, managed futures will generally improve the total risk-adjusted return.

This diversifying characteristic has proven its worth during times of market stress. While other strategies exhibit negative asymmetry during such periods, the right-skewed return distribution of managed futures provides a significant advantage.

LOS 34.f: Discuss investment characteristics, strategy implementation, and role in a portfolio of specialist hedge fund strategies.

Specialist Strategies

Portfolio managers for **specialist hedge fund strategies** use their knowledge of a particular market to pursue niche investment opportunities. The goal of specialist strategies is to generate high risk-adjusted returns that are uncorrelated with those of traditional assets. The risks of such strategies are often unique to the particular niche securities being invested in.

Two such strategies that we will consider in detail are volatility trading and reinsurance/life settlements.

Volatility Trading

Once an esoteric pursuit, **volatility trading** has evolved over recent years to become a recognized investment strategy.

Volatility trading hedge fund managers trade volatility-related assets globally, across countries and across asset classes, in order to exploit perceived differences in volatility prices. The overall goal is to purchase underpriced volatility and sell overpriced volatility.

For example, even though the Tokyo Stock Exchange traditionally has higher volatility than the New York Stock Exchange, implied volatility is usually cheaper in Tokyo than it is in New York. In general, the price of volatility in Asian markets has traditionally been lower than in other regions.

Another volatility trade involves acting as the counterparty to market participants that consistently seek long volatility. Because the negative correlation between stock market returns and equity volatility is high, equity investors seek to buy volatility as a hedge. A hedge fund that is willing to sell volatility will earn an insurance-like premium as compensation for taking on this risk; however, an upturn in volatility can cause such a strategy to unravel in a dramatic fashion.

In the U.S. markets, the most common volatility futures are contracts on the **VIX index**, which tracks the 30-day implied volatility of the S&P 500 index. VIX contracts tend to be mean reverting because high volatility naturally tends to dissipate over time.

Strategy Implementation

Hedge fund managers wishing to pursue volatility trading have several options.

One possibility is to construct various option strategies, such as straddles, calendar spreads, bull spreads, or bear spreads, using basic exchange-traded options.

A second possibility is to make use of over-the-counter (OTC) options, which are customized to meet the portfolio manager's specific needs. A drawback of this method, however, is that it introduces counterparty risk, plus potential liquidity issues.

A more straightforward method of trading volatility is to use futures on the VIX index. An advantage of this approach is that it is a very direct way to express a view on volatility, without the need for hedging. There are some limitations. First, the VIX Index tends to be mean-reverting, which means that trend-following trades tend to be unprofitable. Second, many traders and investors crowd into the VIX futures in order to sell volatility and capture the associated premiums, making it difficult to profit from that strategy.

A fourth method of implementing a volatility trading strategy is to enter into an OTC **volatility swap**, or alternatively a **variance swap**. These derivatives provide a relatively pure exposure to volatility. (Note that the name "swap" here is somewhat misleading. Volatility swaps and variance swaps are actually forward contracts with a payoff based on the difference between observed variance and the expected variance specified in the contract, multiplied by some notional amount.)

Investment Characteristics

The investment characteristics of a volatility trading strategy will vary depending on the securities invested in and the positions taken.

Investors that take a short position in volatility will collect premiums and generally earn stable returns under average market conditions. A long position in volatility will exhibit positive convexity, which can be valuable as a hedge.

The liquidity of a volatility trading strategy will depend on the instruments used. Futures and options based on VIX tend to be extremely liquid, as are exchange-traded volatility options (especially when the tenor is short). On the other hand, OTC contracts are generally less liquid.

The use of futures contracts makes it easy to apply leverage to a volatility trading strategy. The convexity of volatility derivatives means that sometimes large gains can be made from long volatility strategies while taking little risk.

Because of their unique nature, benchmarking of volatility trading strategy performance can be difficult.

Role in a Portfolio

In a portfolio, a long volatility strategy can be a potent diversifier, because stock market volatility is highly negatively correlated with market returns. The disadvantage of maintaining a long volatility position, however, is that a premium must be paid to the volatility seller.

Reinsurance/Life Settlements

In recent years, some hedge funds have sought to take advantage of attractive investment opportunities related to insurance policies. In a typical **life settlement** transaction, an insured person will sell (generally through a broker) their insurance policy to a hedge fund. The hedge fund then will be liable for the premium payments —and will also receive the death benefit upon the passing of the insured.

Individuals sell their life insurance contracts when they feel that they no longer benefit from the agreements. Individuals who purchased life insurance policies may choose to sell their policies to third-party brokers, because those firms will oftentimes pay more for the policy than the issuing insurance company will pay for a surrendered policy.

Catastrophe risk **reinsurance** is another area where hedge funds are increasingly active. **Catastrophe insurance** covers the policy holder against earthquakes, tornadoes, hurricanes, floods, and the like. In order to diversify and decrease risk, insurance companies in their normal course of business will sell off some of their risk to reinsurance companies, who may then resell these risks to hedge funds. Reinsurance can be a rewarding investment for a hedge fund if sufficient diversity can be obtained (i.e., the risks should vary by geography and types of insurance), if the insurance company provides sufficient loan loss reserves, and if the policy premiums are adequate. When considering an investment in catastrophic insurance, a hedge fund evaluates both typical and worst-case outcomes against the insurance premiums to be received. Geographic diversity is important, because a specific catastrophic event will tend to affect only a particular part of the world.

Investment Characteristics

Strategies that involve investments in insurance contracts are illiquid, because insurance policies are somewhat difficult to sell after initiation.

Strategy Implementation

The term "life settlement" refers to a secondary market transaction on an insurance policy. A hedge fund that invests in life settlements will analyze various pools of life insurance contracts that brokers offer, and invest in the ones that they expect to produce an attractive return. After investing, the hedge fund then becomes the beneficiary of these contracts. The investment is successful if the present value of

the future insurance payout exceeds the present value of the payments made by the hedge fund.

In selecting which insurance policies to invest in, a hedge fund will seek out policies with the following characteristics:

- The purchase price of the policy is low.
- The ongoing premium payments are low.
- The insured person is likely to die relatively soon.

One major prerequisite to profiting from life settlements is an accurate alternative estimate of life expectancies. Appraising a life settlement requires a significant amount of skill and knowledge and requires comparing individual policyholders' outlooks to actuarial averages.

Role in a Portfolio

An appealing feature of insurance investments in a portfolio is that the risk inherent in these strategies is almost entirely uncorrelated with market risks and business cycles. For example, floods and earthquakes usually have little correlation to financial markets. Thus, hedge funds that invest in such instruments may be able to increase portfolio alpha, while simultaneously adding return diversification.

LOS 34.g: Discuss investment characteristics, strategy implementation, and role in a portfolio of multi-manager hedge fund strategies.

Multi-Manager Hedge Fund Strategies

Up to this point in the topic review, we have been considering various individual hedge fund strategies. In reality, most investors put money into not just a single hedge fund category, but into a diverse set of strategies.

The notion behind multi-manager hedge funds is to assemble in a deliberate way a portfolio of diverse hedge fund strategies and to adjust the holdings strategically over time.

The most common styles of multi-manager hedge funds are funds-of-funds and multi-strategy funds.

Fund-Of-Funds

A **fund-of-funds (FoF)** takes capital from various individual investors and invests in a number of different hedge funds, generally each pursuing a different strategy.

FoF can provide investors with a number of benefits:

- Diversification across many hedge fund strategies.
- Expertise in individual manager selection.
- Strategic allocation and style allocation.
- Due diligence.
- Occasional value-added tactical decisions.
- Currency hedging.

- Leverage at the portfolio level.
- Better liquidity relative to individual hedge funds.
- Access to certain closed hedge funds.
- Economies of scale in fund monitoring.
- Research expertise.
- Potentially valuable concessions from the underlying funds.

FoF also have the following disadvantages:

- A second layer of fees for the investor.
- Lack of transparency into individual hedge funds.
- No netting of performance fees.
- Additional principal–agent issues.

Investment Characteristics

Individual hedge funds have traditionally employed a "**2 and 20**" fee structure, indicating management fees of 2%, plus performance incentive fees of 20%. On top of these individual fund fees, funds-of-funds have historically added a 1% management fee, plus a further 20% incentive fee on the *total* FoF portfolio. (Though over time, these FoF fees have become negotiable and generally smaller.)

One important benefit that FoF offer is making an investment in hedge funds practical for smaller investors, such as small institutions or moderately wealthy individuals. Individual hedge funds almost always require a significant initial investment; typically $1 million. Achieving a diversified exposure to a number of individual funds poses a difficulty for smaller investors, because an investment in 20 such hedge funds might require a cumulative minimum investment on the order of $20 million. Furthermore, performing due diligence on 20 different hedge funds would require far more resources than most individual investors could muster. (Not to mention complications such as tax reporting on 20 investments.) Using a FoF, on the other hand, a smaller investor can typically access a diversified hedge fund exposure with an investment as small as $100,000. In this way, FoF can serve as an entry point into hedge fund investing.

Aside from providing a straightforward path into hedge funds, FoF provide a number of other advantages. For example, FoF may provide access to high-profile managers whose funds are otherwise closed to new investors. Also, the larger size of an FoF may allow the fund to obtain valuable concessions from the underlying funds' management.

Liquidity can be a challenge for portfolio managers of funds-of-funds. Typically, a FoF will require a one-year initial **lock-up** for investors, and then will allow somewhat greater liquidity afterwards (e.g., monthly or quarterly). However, the underlying funds may have stricter limits on liquidity, potentially putting an FoF manager in a squeeze.

Another drawback of funds-of-funds relates to netting risk: investors could be required to make substantial incentive payments to a small number of successful underlying funds, even if the overall performance of an FoF is poor.

Strategy Implementation

A fund-of-funds strategy is normally implemented as follows:

1. Use fund databases plus personal introductions to become familiar with hedge funds available for investment.
2. Choose an appropriate strategic allocation to different hedge fund strategies.
3. Initiate the manager selection process, applying both top-down and bottom-up techniques.
4. For each hedge fund strategy, consider a number of candidates following that strategy.
5. Interview the candidate hedge fund managers.
6. Review relevant materials such as audit reports.
7. Examine the funds' personnel, operational processes, and risk management.
8. Negotiate with the individual fund managers for lower fees, improved liquidity, or other terms.
9. After the various funds are approved and included in the FoF, an ongoing monitoring process begins, intended to detect major personnel changes, style drift, etc.

The FoF's strategic allocation determines the percentage of total capital that will be invested in each hedge fund style. In addition to this strategic allocation, there may also be a tactical allocation, whereby the FoF manager will at various times underweight or overweight the various hedge fund strategies to reflect the FoF manager's perception of a changing market environment.

Role in a Portfolio

When an FoF manager takes a number of relatively uncorrelated hedge funds and combines them together in the same portfolio, the resulting FoF should produce a number of advantages: greater diversification, steady returns, less concentrated exposure to risks, less volatility, and less exposure to the downside risk of any individual fund manager.

Multi-Strategy Hedge Funds

Like an FoF, **multi-strategy** hedge funds are funds that hold a number of other hedge funds where these various funds are pursuing diverse strategies. Also similar to an FoF, multi-strategy hedge funds are intended to use this diversification of strategy to produce steady, low-volatility returns.

Unlike an FoF, the sub-funds in multi-strategy hedge funds are run by the *same* organization, rather than being managed by different hedge fund firms.

Investment Characteristics

The diversification within multi-strategy funds is intended to produce steady returns and low volatility.

Multi-strategy funds share some investment characteristics with funds-of-funds, but there are also significant differences. For example, the **operational risks** of a multi-

strategy fund are not well-diversified as they are with an FoF, because all of the operational processes of multi-strategy funds are performed under the same roof.

Furthermore, the diversity of strategies represented by the different funds in multi-strategy funds are often somewhat limited, because the managers employed by a specific multi-strategy fund tend to have similar investment viewpoints and methods.

A major advantage of multi-strategy funds over an FoF is the speed and relative ease with which **tactical allocations** can be made. Because each of the multi-strategy funds are managed in-house, it is relatively easy for the multi-strategy manager to reallocate capital from one strategy to another. The high internal transparency and fast response time makes tactical reallocations of multi-strategy funds practical, which could explain why multi-strategy funds have historically been perceived as superior to funds-of-funds for protecting investments.

Investor fees for a multi-strategy fund are often more attractive than those of an FoF. While FoF investors are subject to netting risk (where hefty performance fees can be paid to some successful sub-fund managers, despite overall poor FoF performance), multi-strategy funds are more likely to absorb this netting risk internally. In this arrangement, the investor only pays an incentive fee on the *total* fund performance. (Though some multi-strategy funds do use an FoF-like "pass-through" fee model, which will expose investors to netting risk.)

Like an FoF, multi-strategy funds generally limit investor liquidity using redemption periods and initial lock-ups. Multi-strategy funds often additionally enforce limits on the amount of redemption each quarter.

Strategy Implementation

Multi-strategy funds carry out their approach by making investments in a number of varying hedge fund strategies.

One key advantage of multi-strategy funds is their ability to make tactical reallocations, in addition to the funds' strategic allocation. Furthermore, the multi-strategy fund's internal teams are likely to be well informed about why and when capital and leverage should be reallocated, versus a FoF manager for whom the various funds are more opaque.

Risk management can be more effective with a multi-strategy fund because (unlike FoF managers) multi-strategy managers should have a solid understanding of correlations and common risks between the various funds.

Multi-strategy funds also enjoy efficiencies that come from several hedge fund teams sharing the same administrative resources.

Multi-strategy funds often make greater use of leverage than does the average FoF. Normally, leverage in multi-strategy funds does not pose much of a hazard; however, during periods of market stress, small sources of danger can become significant left-tail risks that threaten the survival of the fund. (This kind of scenario lead to the demise of Ritchie Capital in 2005 and Amaranth Advisors in 2006.)

Multi-strategy funds generally have more varied performance than does an FoF.

Role in a Portfolio

Multi-strategy funds are intended to improve an investment portfolio by adding diversification and steady, low-volatility returns.

Historically, multi-strategy funds have generally performed better than have funds-of-funds, due to a superior fee structure and greater ability to execute on tactical asset allocation. However, the leveraged nature of multi-strategy funds can sometimes lead to a left-tail blow-up during times of stress.

MODULE QUIZ 34.3

1. Considering global macro strategies and managed futures strategies, it would be *most accurate* to state that:
   A. managed futures strategies use more discretionary approaches.
   B. global macro strategies use more systematic approaches.
   C. both strategies tend to be highly liquid and use high leverage.
2. During periods of market stress:
   A. managed futures and global macro both exhibit right-tail skewness.
   B. managed futures strategies exhibit left-tail skewness.
   C. global macro strategies exhibit left-tail skewness.
3. Considering the correlation between equity volatility and equity market returns, the two measures are *most likely* to be:
   A. highly positively correlated.
   B. predominantly uncorrelated.
   C. highly negatively correlated.
4. A hedge fund is *most likely* to purchase a pool of life insurance policies that has high:
   A. surrender value.
   B. ongoing premium payments.
   C. likelihood of the insured person dying soon.
5. Compared to a multi-strategy fund, a fund-of-funds is *most likely* to offer the investor a more:
   A. effective tactical asset allocation.
   B. attractive fee structure.
   C. diverse strategy mix.
6. Compared to a multi-strategy fund, a fund-of-funds is *most likely* to present an investor with higher:
   A. transparency.
   B. netting risks.
   C. leverage.

MODULE 34.4: FACTOR MODELS AND PORTFOLIO IMPACT OF HEDGE FUNDS

Video covering this content is available online.

LOS 34.h: Describe how factor models may be used to understand hedge fund risk exposures.

Factor Models

Analysis of Hedge Fund Strategies

In this LOS, we will consider the use of a **conditional linear factor model** to quantify the risk exposures of various hedge fund strategies.

By *conditional*, we mean a model that takes into account that a fund may behave one way during normal market conditions, but perform differently during a period of market turbulence (such as the economic crisis of 2007–2009).

By determining a fund's actual risk exposures, events such as the widespread closure of hedge funds that happened after the global financial crisis are less likely.

Conditional Factor Risk Model

Suppose we use the following conditional linear factor model to explain the return of hedge fund *i* in period *t*:

$$\begin{aligned}(\text{Return on HF}_i)_t = {} & \alpha_i + \beta_{i,1}(\text{Factor 1})_t + \beta_{i,2}(\text{Factor 2})_t + \ldots + \beta_{i,K}(\text{Factor K})_t + \\ & D_t\beta_{i,1}(\text{Factor 1})_t + D_t\beta_{i,2}(\text{Factor 2})_t + \ldots + D_t\beta_{i,K}(\text{Factor K})_t \\ & + (\text{error})_{i,t}\end{aligned}$$

where:

α_i	= intercept for hedge fund *i*
$\beta_{i,K}(\text{Factor K})_t$	= exposure during *normal* periods to risk factor *K*
D_t	= dummy variable that equals zero during normal periods, and one during a financial crisis
$D_t\beta_{i,K}(\text{Factor K})_t$	= *incremental* exposure to risk factor *K* during financial crisis periods
$(\text{error})_{i,t}$	= random error with zero mean

Any returns not explained by the model's risk factors would be attributed to either omitted risk factors, alpha (i.e., hedge fund manager skill), or randomness (error).

Hasanhodzic and Lo (2007) used the following six factors:

1. *Equity risk (SNP500):* S&P 500 total return index.
2. *Interest rate risk (BOND):* Bloomberg Barclays Corporate AA Intermediate Bond Index.
3. *Currency risk (USD):* U.S. Dollar Index.
4. *Commodity risk (CMDTY):* Goldman Sachs Commodity Index (GSCI) total return.
5. *Credit risk (CREDIT):* Spread between Moody's Baa and Aaa corporate bond yields.
6. *Volatility risk (VIX):* CBOE Volatility Index (VIX).

A **stepwise regression** process is useful for creating linear conditional factor models that avoid multicollinearity problems, because it avoids the use of highly correlated risk factors. When this stepwise regression process was run by the authors of the original reading, the process resulted in the BOND and CMDTY factors being dropped due to multicollinearity issues. (CREDIT and SNP500 respectively produced higher adjusted R^2.)

This left the following four factors for measuring risk exposures:

1. Equity risk (SNP500).
2. Currency risk (USD).
3. Credit risk (CREDIT).
4. Volatility risk (VIX).

Each hedge fund strategy has different exposures to these various risk factors. These risk factors stem from taking long or short positions in financial instruments that are exposed to these risks.

For example, arbitrage strategies are often exposed to credit spread risk and market volatility risk. Event-driven strategies and L/S equity strategies generally have significant exposure to equity (market beta) risk.

PROFESSOR'S NOTE

The curriculum variously refers to this conditional factor risk model as a "factor model," "linear factor model," "conditional risk model," etc. These terms are all intended to refer to the same idea.

LOS 34.i: Evaluate the impact of an allocation to a hedge fund strategy in a traditional investment portfolio.

We will assume a 20% allocation to a hedge fund strategy from a conventional portfolio comprised of 60% stock and 40% bond. After the allocation, the new weights will be 20% hedge fund, 48% stock, and 32% bond.

Portfolio Contribution of Hedge Fund Strategies

Performance Contribution to a 60/40 Portfolio

For most strategies, when added to a conventional portfolio:

- Total portfolio standard deviation decreases.
- Sharpe ratio increases.
- Sortino ratio increases.
- Maximum drawdown decreases (in approximately one-third of portfolios).

The interpretation of these results is that hedge fund strategies generally increase risk-adjusted return and provide diversification to a traditional portfolio of stocks and bonds.

Risk-Adjusted Performance

The **Sharpe ratio** is one risk-adjusted measure of performance. The risk metric used to calculate the Sharpe ratio is standard deviation—with the result that both downside *and upside* deviations result in a lower Sharpe ratio.

The **Sortino ratio** is a similar risk-adjusted measure of performance; however, only *downside* deviations are considered to reflect risk: risk is measured as variability *below* a predefined level of return. Because of the left-tail risk present in many hedge fund strategies, the Sortino ratio is generally seen as a superior measure of the risk-adjusted performance of hedge funds.

When 20% allocations to various hedge fund strategies are added to the traditional stock/bond portfolio, particularly high Sharpe ratios are achieved from allocations to these strategies:

- Systematic futures.

- Distressed securities.
- Fixed-income arbitrage.
- Global macro.
- Equity market neutral.

The highest Sortino ratios were attained via allocations to the following hedge fund strategies:

- Equity market neutral.
- Systematic futures.
- Long/short equity.
- Event driven.

Allocations to the following strategies were found to be effective in generating superior risk-adjusted performance, based on comparatively high Sharpe and Sortino ratios:

- Systematic futures.
- Equity market neutral.
- Global macro.
- Event-driven hedge fund strategies.

On the other hand, it was observed that the following fund strategies do not significantly enhance risk-adjusted performance:

- Fund-of-funds.
- Multi-strategy.

While all of the hedge fund strategies expose the overall portfolio to various kinds of additional portfolio risks, allocating a portion of a stock/bond portfolio to hedge funds generally reduces risk and increases returns.

Risk Metrics

One key reason for investors to allocate a portion of their portfolio to hedge fund strategies is to reduce risk.

Standard Deviation

Perhaps not surprisingly, it was found that the following strategies resulted in the lowest standard deviations of returns for the overall portfolio:

- Dedicated short-biased.
- Bear market neutral.

These funds also produced notably low standard deviations:

- Systematic futures.
- FoF: macro/systematic.
- Equity market neutral.

The risk-reduction ability of these strategies has been found to be substantial. Not surprisingly, they are also some of the strategies that most improve risk-adjusted

returns.

Funds that were found to have little positive impact on reducing standard deviations of the overall portfolio include:

- Event-driven: distressed securities.
- Relative value: convertible arbitrage.

An explanation for event-driven: distressed securities' lack of ability to reduce standard deviation is that these funds tend to take long positions in securities, and outcomes are either mild successes or grand failures.

Relative value: convertible arbitrage funds do little to improve standard deviation, likely because this strategy's leveraged nature becomes a liability during times of market volatility.

Drawdown

Drawdown is defined as the peak-to-trough decline for a portfolio, generally quoted as the percentage drop between a point of high portfolio value and the subsequent trough. The **high-water mark** refers to the maximum value the portfolio has *ever* reached.

Drawdown is useful for quantifying the risk and past performance of various investment strategies, and for comparing one strategy's risk to another.

The hedge fund strategies that produced the smallest maximum drawdowns are various opportunistic strategies:

- Global macro.
- Systematic futures.
- Merger arbitrage.
- Equity market neutral.

Use of the conditional risk model can show that these strategies perform relatively well during periods of market crisis because they have minimal exposure to credit risk or equity. Furthermore, these strategies benefit from their liquid nature. Those properties make opportunistic strategies useful diversifiers for traditional assets.

Other hedge fund strategies did little to mitigate the traditional portfolio's maximum drawdown:

- Long/short equity.
- Event-driven: distressed securities.
- Relative value: convertible arbitrage.

These results are somewhat expected. Using the conditional risk model, we can demonstrate that these particular strategies have significant exposure to equity risk, and furthermore during crisis periods they have significant exposure to credit risk as well.

MODULE QUIZ 34.4

1. Conditional linear factor models used to understand hedge fund risk exposures are *most likely* to use factors that include:

A. liquidity risk, operational risk, valuation risk, and systemic risk.
B. interest rate risk, commodity risk, margining risk, and concentration risk.
C. equity risk, credit risk, currency risk, and volatility risk.

2. Adding a 20% allocation of a hedge fund strategy to a traditional 60%/40% portfolio is *most likely* to increase the total portfolio's:
A. standard deviation.
B. maximum drawdown.
C. Sortino ratio.

3. The risk-adjusted performance of a traditional 60%/40% portfolio is *most likely* to be improved by adding an allocation to a hedge fund using the strategy of:
A. equity market neutral.
B. fund-of-funds.
C. multi-strategy.

KEY CONCEPTS

LOS 34.a

Hedge fund strategies are classified according to the instruments they invest in, the philosophy followed, and the kinds of risk exposures taken.

This reading classifies hedge fund strategies into the following six categories:

1. Equity related.
2. Event driven.
3. Relative value.
4. Opportunistic.
5. Specialist.
6. Multi-manager.

LOS 34.b

Long/short equity. This strategy generates alpha via careful stock picking. L/S funds are typically liquid, and generally net long. Equity L/S managers aspire to the returns of a long-only approach, but with lower standard deviations. The more market-neutral the approach, the more leverage is likely to be applied.

Dedicated short-selling and short-biased strategies. These two strategies have negative correlations to traditional assets and modest return goals. The focus is on stock picking, using minimal leverage. Dedicated short strategies are generally 60% to 120% short at all times. Short-biased strategies are typically 30% to 60% net short: the short beta exposure is moderated with some long exposure (and cash).

Equity market-neutral (EMN). These strategies attempt to profit from short-term mispricing between securities. Beta risk is minimal, meaning that EMN strategies can perform well in periods of market weakness. Most managers are quantitative (vs. discretionary). High leverage is usually used.

LOS 34.c

Merger arbitrage. This strategy attempts to profit by taking positions on a corporate takeover. Merger arbitrage returns are usually insurance-like, with a high Sharpe ratio. However, left-tail risk is present: negative returns can occur if a merger

deal unexpectedly fails. Some leverage is usually applied to generate meaningful returns. Merger arbitrage is a relatively liquid strategy.

Distressed securities. These strategies seek out mispriced securities of firms in bankruptcy or facing other financial stress. Distressed securities strategies are usually long biased, with high illiquidity and moderate or low leverage. Returns tend to be high compared to those of other event-driven strategies.

LOS 34.d

Fixed-income arbitrage. This strategy attempts to profit from the mispricing of bonds. Sub-strategies include yield curve trades and carry trades. Fixed-income arbitrage usually uses high leverage.

Convertible arbitrage. These strategies attempt to extract "underpriced" implied volatility from long convertible bonds. Convertible arbitrage works best when there is high convertible issuance, adequate market liquidity, and moderate volatility. Liquidity issues may arise from convertibles being somewhat difficult to convert to cash. Convertible arbitrage managers typically run about 300% long and 200% short.

LOS 34.e

Opportunistic strategies tend to be highly liquid and use high leverage.

Global macro. These strategies use discretionary approaches and a range of financial instruments to exploit trends in capital markets across the world. Turbulent global markets can represent both a risk and an opportunity for global macro funds; hence the returns of these strategies can be volatile and lumpy.

Managed futures. In these strategies, a portfolio of futures contracts is actively managed using systematic approaches to provide portfolio and market diversification. Managed futures strategies often exhibit right-tail skew during market turmoil.

LOS 34.f

Specialist hedge fund strategies operate in market niches in order to generate uncorrelated returns. Success with these strategies usually requires specialized knowledge.

Volatility traders. These strategies seek to profit from changes in the term structure of volatility. OTC options can be used to create bull spreads, bear spreads, straddles, and calendar spreads. Alternatively, other instruments including VIX futures, volatility swaps, and variance swaps can be used.

Life settlements. In these strategies, pools of life insurance contracts are purchased, and the hedge fund becomes the beneficiary. The hedge fund manager looks for policies with low surrender value, low ongoing premium payments, and high probability that the insured person will die soon.

LOS 34.g

Multi-manager hedge fund approaches use strategy diversification in an attempt to produce low-volatility, steady returns.

Funds-of-funds. This strategy comprises a hedge fund that invests in other hedge funds. Funds-of-funds can offer a very broad strategy mix but can suffer from a lack of transparency, slower tactical execution, and can also expose the FoF investor to netting risk.

Multi-strategy funds. In this hedge fund approach, a single hedge fund pursues a combination of strategies all under one roof. Compared to funds-of-funds, multi-strategy funds offer a better fee structure and faster tactical asset allocation, though operational risks are less diversified.

LOS 34.h

Conditional linear factor models can be useful for analyzing hedge fund strategies in terms of their risk factor exposures. The curriculum makes use of a specific four-factor model (incorporating equity risk, currency risk, volatility risk, and credit risk factors) to quantify a strategy's exposures.

LOS 34.i

Hedge funds generally bring diversification to traditional stock/bond portfolios, and enhance risk-adjusted returns. The addition of a 20% hedge fund allocation to a traditional 60% stock/40% bond portfolio generally decreases the portfolio's total standard deviation, increases the Sharpe and Sortino ratios, and decreases maximum drawdown.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 34.1

1. **C** Convertible bond arbitrage strategies are generally classified as relative value strategies. (LOS 34.a)

2. **A** Managed futures strategies are generally classified as opportunistic strategies. (LOS 34.a)

Module Quiz 34.2

1. **A** EMN strategies usually apply somewhat high levels of leverage in order to produce meaningful levels of return. Neither dedicated short strategies nor short-biased strategies typically make significant use of leverage. (LOS 34.b)

2. **C** Equity long/short strategies typically have gross exposures of 70% to 90% long and 20% to 50% short. Dedicated short strategies are usually 60% to 120% short at all times. Short-biased strategies are usually around 30% to 60% net short. (LOS 34.b)

3. **A** Compared to various other hedge fund approaches, EMN strategies generally have relatively modest returns. EMN funds' primary source of return is alpha. They do not take on beta risk. Their lack of market exposure make EMN strategies attractive in periods of market weakness. (LOS 34.b)

4. **A** While short positions are possible in distressed securities investing, this strategy is usually long biased. Illiquidity tends to be high, and the strategy generally uses moderate to low leverage. (LOS 34.c)

5. **A** When a firm's assets are sold off in liquidation, securities holders are paid sequentially depending on the priority of their claims: first senior secured debt, then junior secured debt, unsecured debt, convertible debt, preferred stock, and lastly common stock. (LOS 34.c)

6. **A** Convertible arbitrage managers typically attempt to extract underpriced implied volatility from holdings of long convertible bonds. To delta and gamma hedge these exposures, the managers will take *short equity* positions. (LOS 34.d)

Module Quiz 34.3

1. **C** Managed futures strategies are usually implemented via systematic approaches, while global macro strategies more often use discretionary approaches. Both strategies typically use high leverage and tend to be highly liquid. (LOS 34.e)

2. **A** Returns of managed futures and global macro strategies both typically exhibit right-tail (positive) skewness during times of market stress. Global macro strategies, however, generally deliver more heterogeneous outcomes. (LOS 34.e)

3. **C** Equity volatility is roughly 80% negatively correlated with equity market returns: volatility levels rise when equity markets fall. This characteristic makes long volatility strategies useful diversifiers for long equity investments. (LOS 34.f)

4. **C** In implementing life settlement strategies, a hedge fund manager looks for policies with the following traits: low surrender value being offered to the insured individual, low ongoing premium payments required of the investor, and high probability that the insured person will die sooner than predicted by actuarial methods. (LOS 34.f)

5. **C** Funds-of-funds generally offer a more diverse strategy mix than do multi-strategy funds. Multi-strategy funds offer *quicker* tactical asset allocation and generally a *better* fee structure (for example, netting risk between strategies is often absorbed by the multi-strategy general partner). (LOS 34.g)

6. **B** Compared to multi-strategy funds, funds-of-funds offer an investor less transparency and higher netting risk. Multi-strategy funds exhibit higher variance due to less diverse strategies and use comparatively higher leverage. (LOS 34.g)

Module Quiz 34.4

1. **C** This reading uses a model that incorporates four factors: equity risk, credit risk, currency risk, and volatility risk. (The interest rate risk "BOND" and commodity risk "CMDTY" factors used by Hasanhodzic and Lo were dropped due to multicollinearity issues.) (LOS 34.h)

2. **C** Adding a 20% hedge fund allocation to a traditional 60%/40% portfolio usually *decreases* total portfolio standard deviation while it increases Sharpe

and Sortino ratios in the combined portfolios. An allocation to hedge funds often *decreases* maximum drawdown. (LOS 34.i)

3. **A** Adding an allocation to equity market-neutral hedge fund strategies to a traditional portfolio has been shown to be effective in generating superior risk-adjusted performance, as evidenced by high Sharpe and Sortino ratios. (The same is true of systematic futures, global macro, and event-driven strategies.) On the other hand, fund-of-funds and multi-strategy funds have been found not to enhance risk-adjusted performance significantly. (LOS 34.i)

Topic Quiz: Alternative Investments

You have now finished the Alternative Investments topic section. Please log into your Schweser online dashboard and take the Topic Quiz on this section. The Topic Quiz provides immediate feedback on how effective your study has been for this material. Questions are more exam-like than typical Module Quiz or QBank questions; a score of less than 70% indicates that your study likely needs improvement. These tests are best taken timed; allow three minutes per question.

FORMULAS

Fixed Income

price of a T-period zero-coupon bond

$$P_T = \frac{1}{(1 + S_T)^T}$$

forward price (at t = j) of a zero-coupon bond maturing at (j+k)

$$F_{(j,k)} = \frac{1}{[1 + f(j,k)]^k}$$

forward pricing model

$$P_{(j+k)} = P_j F_{(j,k)}$$

Therefore:

$$F_{(j,k)} = \frac{P_{(j+k)}}{P_j}$$

forward rate model

$$[1 + S_{(j+k)}]^{(j+k)} = (1 + S_j)^j \, [1 + f(j,k)]^k$$

or

$$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$$

swap spread

$$\text{swap spread}_t = \text{swap rate}_t - \text{Treasury yield}_t$$

TED spread

TED spread = (3-month MRR) – (3-month T-bill rate)

MRR-OIS spread

MRR-OIS spread = MRR – "overnight indexed swap" rate

portfolio value change due to level, steepness, and curvature movements

$$\frac{\Delta P}{P} \approx -D_L \Delta x_L - D_S \Delta x_S - D_C \Delta x_C$$

callable bond

$$V_{call} = V_{straight} - V_{callable}$$

putable bond

$$V_{putable} = V_{straight} + V_{put}$$

$$V_{put} = V_{putable} - V_{straight}$$

$$\text{effective duration} = ED = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$$

$$\text{effective convexity} = EC = \frac{BV_{-\Delta y} + BV_{+\Delta y} - (2 \times BV_0)}{BV_0 \times \Delta y^2}$$

convertible bond

minimum value of convertible bond
= greater of conversion value or straight value

$$\text{market conversion price} = \frac{\text{market price of convertible bond}}{\text{conversion ratio}}$$

market conversion premium per share
= market conversion price – stock's market price

$$\text{market conversion premium ratio} = \frac{\text{market conversion premium per share}}{\text{market price of common stock}}$$

$$\text{premium over straight value} = \left(\frac{\text{market price of convertible bond}}{\text{straight value}}\right) - 1$$

callable and putable convertible bond value = straight value of bond
+ value of call option on stock
− value of call option on bond
+ value of put option on bond

credit analysis

recovery rate = percentage of money received upon default of the issuer

loss given default (%) = 100 – recovery rate

expected loss = probability of default × loss given default

present value of expected loss
= (value of risk-free bond) – (value of credit-risky bond)

upfront premium % (paid by protection buyer)
≈ (CDS spread – CDS coupon) × duration

price of CDS (per $100 notional) ≈ $100 – upfront premium (%)

profit for protection buyer ≈ change in spread × duration × notional principal

Derivatives

forward contract price (cost-of-carry model)

$$FP = S_0 \times (1 + R_f)^T$$

or

$$S_0 = \frac{FP}{(1 + R_f)^T}$$

no-arbitrage price of an equity forward contract with discrete dividends

$$FP(\text{on an equity security}) = (S_0 - PVD) \times (1 + R_f)^T$$

$$FP(\text{on an equity security}) = \left[S_0 \times (1 + R_f)^T\right] - FVD$$

value of the long position in a forward contract on a dividend-paying stock

$$V_t(\text{long position}) = [S_t - PVD_t] - \left[\frac{FP}{(1+R_f)^{(T-t)}}\right]$$

price of an equity index forward contract with continuous dividends

$$FP(\text{on an equity index}) = S_0 \times e^{(R_f^c - \delta^c)\times T} = (S_0 \times e^{-\delta^c \times T}) \times e^{R_f^c \times T}$$

where:

R_f^c = continuously compounded risk-free rate

δ^c = continuously compounded dividend yield

forward price on a coupon-paying bond

$$FP(\text{on a fixed income security}) = (S_0 - PVC) \times (1+R_f)^T$$

or

$$= S_0 \times (1+R_f)^T - FVC$$

value prior to expiration of a forward contract on a coupon-paying bond

$$V_t(\text{long position}) = [S_t - PVC_t] - \left[\frac{FP}{(1+R_f)^{(T-t)}}\right]$$

price of a bond futures contract

$$FP = [(\text{full price})(1+R_f)^T - FVC - AI_T]$$

quoted bond futures price based on conversion factor (CF)

$$QFP = FP/CF = [(\text{full price})(1+R_f)^T - FVC - AI_T]\left(\frac{1}{CF}\right)$$

swap fixed rate

$$SFR(\text{periodic}) = \frac{1 - \text{final discount factor}}{\text{sum of discount factors}}$$

swap fixed rate(annual)
= SFR(periodic) × number of settlements periods per year

value of plain vanilla interest rate swap (to payer) after inception

$$\text{value to the payer} = \sum DF \times (SFR_{New} - SFR_{Old}) \times \frac{\text{days}}{360} \times \text{notional principal}$$

probability of an up-move or down-move in a binomial stock tree

$$\pi_U = \text{probability of an up move} = \pi_U = \frac{1 + R_f - D}{U - D}$$

$$\pi_D = \text{probability of a down move} = (1 - \pi_U)$$

put-call parity

$$S_0 + P_0 = C_0 + PV(X)$$

put-call parity when the stock pays dividends

$$P_0 + S_0 e^{-\delta T} = C_0 + e^{-rT}X$$

dynamic hedging

$$\text{number of short call options needed to delta hedge} = \frac{\text{number of shares hedged}}{\text{delta of call option}}$$

$$\text{number of long put options needed to delta hedge} = -\frac{\text{number of shares}}{\text{delta of the put option}}$$

change in option value

$$\Delta C \approx \text{call delta} \times \Delta S + \tfrac{1}{2}\,\text{gamma} \times \Delta S^2$$

$$\Delta P \approx \text{put delta} \times \Delta S + \tfrac{1}{2}\,\text{gamma} \times \Delta S^2$$

option value using arbitrage-free pricing portfolio

$$C_0 = hS_0 + \frac{(-hS^- + C^-)}{(1+R_f)} = hS_0 + \frac{(-hS^+ + C^+)}{(1+R_f)}$$

$$P_0 = hS_0 + \frac{(-hS^- + P^-)}{(1+R_f)} = hS_0 + \frac{(-hS^+ + P^+)}{(1+R_f)}$$

BSM model

$$C_0 = S_0 e^{-\delta T} N(d_1) - e^{-rT} X N(d_2)$$

$$P_0 = e^{-rT} X N(-d_2) - S_0 e^{-\delta T} N(-d_1)$$

Alternative Investments

Theory of Storage

commodity futures price = spot price + storage costs – convenience yield

NCREIF Property Index (NPI) calculation

$$\text{return} = \frac{\text{NOI} - \text{capital expenditures} + (\text{end market value} - \text{beg market value})}{\text{beginning market value}}$$

net asset value approach to REIT share valuation

estimated cash NOI
÷ assumed cap rate
= estimated value of operating real estate
+ cash and accounts receivable
– debt and other liabilities
= net asset value
÷ shares outstanding
= NAV/share

price-to-FFO approach to REIT share valuation

funds from operations (FFO)
÷ shares outstanding
= FFO/share
× sector average P/FFO multiple
= NAV/share

price-to-AFFO approach to REIT share valuation

funds from operations (FFO)
– non-cash rents:
– recurring maintenance-type capital expenditures
= AFFO
÷ shares outstanding
= AFFO/share
× property subsector average P/AFFO multiple
= NAV/share

discounted cash flow approach to REIT share valuation

value of a REIT share

= PV(dividends for Years 1 through n) + PV(terminal value at the end of Year n)

conditional factor risk model:

$$(\text{Return on HF}_i)_t = \alpha_i + \beta_{i,1}(\text{Factor 1})_t + \beta_{i,2}(\text{Factor 2})_t + \ldots + \beta_{i,K}(\text{Factor K})_t + D_t\beta_{i,1}(\text{Factor 1})_t + D_t\beta_{i,2}(\text{Factor 2})_t + \ldots + D_t\beta_{i,K}(\text{Factor K})_t + (\text{error})_{i,t}$$

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